Bauhaus-Universität Weimar

Quality Assessment Methods for Experimental Models in Structural Engineering

(Methoden der Qualitätsbewertung für experimentelle Modelle im Konstruktiven Ingenieurbau)

DISSERTATION

zur Erlangung des akademischen Grades
Doktor-Ingenieur (Dr.-Ing.)

an der Fakultät Bauingenieurwesen
der Bauhaus-Universität Weimar

vorgelegt von

Hem Bahadur Motra

geboren am 26. April 1981
in Gulmi Nepal

Gutachter: Prof. Dr.-Ing. Jörg Hildebrand
Prof. Dr.-Ing. Hamid Sadegh-Azar
Prof. Dr.-Ing. habil. Frank Wuttke

Tag der Disputation: 26. Juni 2015

To my father Rudra Bahadur, mother Lalmati and wife Sarita

Acknowledgments

The present work was written during my work as a research associate within the Research Training Group 1462 "Assessment of Coupled Experimental and Numerical Partial Models in Structural Engineering" at the Bauhaus-Universität Weimar supported by the German Research Institute (DFG) which is gratefully acknowledged.

I express my sincere gratitude to my respected supervisor Prof. Dr.-Ing. Jörg Hildebrand, Department of Civil Engineering, Bauhaus Universität Weimar, for his continuous guidance, valuable suggestions and the support given to me during all the stages of this thesis. His constant encouragement and the constructive remarks he use to give me during our regular meetings helped me a lot to understand the subject and has also motivated me to carry out research work in the area of uncertainty quantification. I am most thankful to Prof. Dr.-Ing. Andrea Osburg, whose encouragement, guidance and support from the initial to final stages enabled me to complete my research work. I appreciate her contributions of time, ideas, and openness in discussions.

I would also like to thank Prof. Dr. rer. nat. Tom Lahmer for his assistance during my work. I am also indebted to Dr.-Ing. Holger Keitel for his support and the discussions related to the assessment problem. I also appreciate the co-operation and support provided by the faculty and the colleagues of GRK 1462, Bauhaus-Universität Weimar.
I am also thankful for Prof. Dr.-Ing. habil. Frank Wuttke and Prof. Dr.-Ing. Hamid Sadegh-Azar for agreeing to serve as examiners for my dissertation and taking the time to review my work.

I am deeply indebted to my family for supporting me throughout my career. My wife has been a constant source of support and she has stood by me during all the difficult situations throughout these years. I extend my heartfelt thanks to my brothers, sister, and their families for always supporting and encouraging me. The constant prayers and blessings of my parents have given me the strength to do my best. I sincerely acknowledge the good wishes of my friends, colleagues, and well-wishers.

Hem Bahadur Motra

Weimar, 2015

Abstract

Experimental and numerical models are required in order to reliably assess the safety and usability of both newly constructed and existing structures. The quality of both numerical and experimental models must be evaluated in order to reliably predict structural behaviour and design. Many statements about the quality of a simulation model can only be validated by including the appropriate experiments e.g. the quantification of the statistical uncertainties of model input parameters during the calibration of the confidence level estimator model, which is dependent heavily on the definition of the experiment and the quality of its implementation. Metrological aspects should therefore be used in order to guarantee the equivalence of results between different laboratories and evaluate the measurement or simulation result with its specifications. However, methodology for quantitatively assessing the implementation and results of experimental models is lacking..

This work presents methods for assessing the quality of different materials used in structural engineering and monitoring models. Statistical models and methods of statistical inference provide the technical machinery necessary to evaluate the properties of models including uncertainty, sensitivity, reliability, robustness, and complexity. Furthermore, the quality of the experimental models evaluated is based on these properties. The concepts of total uncertainty and reliability are employed in order to rank the experimental models studied in order to determine the properties of the materials. Some of these models and methods are illustrated by (i) measuring the mechanical properties of structural steel using tensile testing, (ii) measuring the properties of PCC concrete using compressive testing, (iii) measuring soil properties using the triaxial test, and (iv) the monitoring models of concrete poles. The methodology presented in this study provides the mathematical and computational tools required for quantifying the quality and uncertainty of the experimental models. This can be used to improve measurement processes and promote quality and capacity with respect to decision-making.

Zusammenfassung

Für eine zuverlässige Beurteilung der Sicherheit und Gebrauchstauglichkeit von neu zu errichtenden und bestehenden Strukturen sind experimentelle und numerische Modelle erforderlich. Die Qualitätsbewertung von numerischen und experimentellen Modellen ist heutzutage zur zuverlässigen Vorhersage des Strukturverhaltens und in der Planung von Bauwerken unabkömmlich. Viele Aussagen über die Qualität eines Simulationsmodells lassen sich nur mittels geeigneter Experimente treffen, wie beispielsweise die Quantifizierung von statistischen Unsicherheiten in Modelleingangsparametern. Basierend auf der Modellkalibrierung mittels Konfidenzniveausschätzung, sind diese allerdings stark abhängig von der Definition des Experiments und der Qualität der Durchführung. Folglich wird der Einsatz messtechnischer Aspekte notwendig, um die Gleichwertigkeit der Ergebnisse zwischen verschiedenen Laboratorien, als auch der Auswertung von Messungen oder Simulationsergebnissen mit ihren jeweiligen Spezifikationen, sicherzustellen. Es besteht derzeit jedoch noch ein Defizit an Methoden zur quantitativen Beurteilung der Qualität der Ergebnisse von Versuchsmodellen und deren beispielhafter Implementierung.

Daher werden in dieser Arbeit Methoden zur Qualitätsbeurteilung von experimentellen Modellen für verschiedene Materialien im Konstruktiven Ingenieurbau präsentiert. Statistische Modelle und Methoden der statistischen Inferenz bieten die technischen Möglichkeiten, Modelleigenschaften, wie Unsicherheit, Sensitivität, Zuverlässigkeit, Robustheit und Komplexität, zu bewerten. Darüber hinaus wird die Qualität der experimentellen Modelle aufgrund dieser Modelleigenschaften bewertet. Die Konzepte der Gesamtunsicherheit und des Zuverlässigkeitsindex werden eingesetzt, um die experimentellen Modelle zur Bestimmung von Materialeigenschaften zu bewerten. Einige dieser Modelle und Verfahren werden in den folgenden vier Beispielen näher erläutert: (i) Messung der mechanischen Eigenschaften von Baustahl mittels Zugversuchs; (ii) Messung der Materialeigenschaften von PCC-Beton mittels Druckversuchs; (iii) Messung von Bodeneigenschaften mittels Triaxialversuchs; iv) Monitoringmodell eines Betonmastes. Als Ergebnis der Studie kann festgestellt werden, dass die mathematischen und algorithmischen Werkzeuge in der Lage sind, Modellunsicherheiten und die experimentelle Modellqualität zu quantifizieren. Darüber hinaus ist es sinnvoll, Messverfahren zu verbessern, um die Qualität und Kapazität der Entscheidungsfindung zu fördern.

Contents

List of Figures

List of Tables

Nomenclature

Nomenclature not listed in the following tables will be explained in the text.

Abbreviations

ACI	american concrete institute
BDM	boundary element method
BR	base result
CV	coefficient of variance
CC	normal concrete
FDM	finite difference method
FEM	finite element method
EM	experimental models
EMQA	experimental model quality assessment
GDP	gross domestic product
ICC	interclass correlation
ISO	international organisation for standardisation
GUM	guide to the expression of uncertainties in measurement
LPU	law of propagation of uncertainty
MCM	Monte Carlo method
MU	measurement uncertainty
MV	measured value
NIST	national institute of standards and technology
OAT	one at a time
PDF	probability density function
PM	partial model
SA	sensitivity analysis
SCC	polymer modified concrete
SEM	standard error of measurement
SM	measured system

Nomenclature

A	cross sectional area
c_i	sensitivity coefficient
C	sensitivity coefficient matrix
$Corr(X_i; X_j)$	correlation for two random variables, X_i and X_j
$Cov(X_i; X_j)$	covariance for two random variables, X_i and X_j
$det(J)$	Jacobian determinant
$E(X_i)$	expectation of random variable X_i
G	discrete representation of distribution function $G_Y(\eta)$
$G_X(\xi)$	distribution function with variable ξ for input quantity X
$g_{X_i}(\xi_i)$	probability density function with variable ξ for input quantity X_i
$G_Y(\eta)$	distribution function with variable η for output quantity Y
$g_Y(\eta)$	probability density function with variable η for output quantity Y
$\frac{\partial f}{\partial x_i}$	partial derivative with respect to input quantity X_i
h	univariate measurement model expressed as a relationship between X and Y
M	number of trials in MCM
$r(x_i, x_j)$	coefficient of correlation
n	independent observations
N	input quantities
p	probability; level of confidence
$s(\bar{X}_i)$	experimental standard deviation of the mean $(\bar{X})$
$s(\bar{X}_i, \bar{X}_j)$	estimate of the covariance of input means $(\bar{X}_i)$ and $(\bar{X}_i)$
$\bar{X}_i$	mean of sample
X_i	input quantities
$X_{i,k}$	series of observations
X_{low}	lower level of confidence interval
X_{high}	upper level of confidence interval
Y_i	output quantities
β	reliability index
σ	standard deviation
ν_{eff}	effective degree of freedom
μ_A	type A standard uncertainty
μ_B	type B standard uncertainty

E	Young's modulus
F	force
l	gauge length
l_0	distance between the stain measured
b	width of specimen
t	thickness of specimen
ϵ	strain
$K_{m.norm}$	standardized complexity
$\eta_{PM,m}$	number of partial model
$\eta_{PM,max}$	number of partial models of the most complex
p_f	probability of failure
R	resistance of structures
S	sensitivity index
S_i	first-order sensitivity index
$S_{i,j}$	second-order sensitivity index
S_T	total sensitivity index
S_i^{GUM}	local sensitivity coefficients
S/N	signal-to-noise ratio
$\bar{y}$	mean of observed data
s_y^2	variance of y
S_{home}	standard deviation of the homogeneity test
n_{home}	the number of tested/used samples for homogeneity test
$z - score$	standard value of proficiency test
η	noise ratio for robust design
$\mu(x)$	standard uncertainty of measurement input
$\mu(y)$	combined standard uncertainty of measurement output
μ_R	mean of resistance
μ_S	mean of effect
σ_R	standard deviation of resistance
ζ	standard uncertainty of the participants result
σ_S	standard deviation of effect
$\dot{e}_{L_c}$	speed rate
U_E	total measurement uncertainty
U_x	uncertainty matrix

ΔH	height variation
ΔV	volume variation
f_c	compressive strength of concrete
p	mean stress
p'	mean effective stress
q	deviator stress
q'	effective deviator stress
σ_1	axial stress
$\sigma_1^{'}$	effective axial stress
σ_3	horizontal stress
$\sigma_3^{'}$	effective horizontal stress
ϕ'	friction angle
c'	cohesion
U_{ref}	uncertainty of reference laboratory
$V(X)$	covariance matrix of random variable X
R_{eH}	upper yield strength
R_{eL}	lower yield strength
μ_{rep}	uncertainty because of repeatability
μ	pore pressure
E_{mech}	mechanical strain
E_{amp}	amplifier strain
E_{app}	apparent strain
$\theta_{E_{mech}}$	model uncertainty
$f_{s,q}$	transverse strain correction factor
$\alpha_{s,k}$	temperature coefficient
$f_{s,v}$	gauge variation factor
$f_{a,a}$	amplifier deviation
$f_{s,s}$	model uncertainty of the gauge factor variation
$f_{a,z}$	zero deviation
ν	Poisson's ratio
q	transverse sensitivity coefficient
β_G	temperature coefficient of resistance
α_S	thermal expansion coefficients of the substrate
α_G	thermal expansion coefficients of the grid

ΔT	temperature difference
$P_{\mu,i}$	prior probability
$\epsilon_{R,j}$	reference strain
$\theta_{\epsilon mech}$	model uncertainty for observation equation
ε_{mech}	mechanical strain for observation equation
ε_{amp}	amplifier strain for observation equation
ε_{app}	apparent strain for observation equation

Chapter 1

Introduction

1.1 Background and Motivation

Experimental and mathematical/numerical models are required for the reliable assessment of safety and usability in both newly constructed and existing structures. For this purpose, in any engineering discipline – including structural engineering – current research entails the simultaneous development of models and implementation of experiments. Structural engineering covers a broad spectrum of domains in which the experimental component often plays an important role. Computer simulation, which has become an important tool in the design of civil engineering structures, utilises physical and mathematical models. This allows the structural components to be designed to specification. However, simulation models are only approximations of reality; the models must be tested in practice. Numerical models must be validated with experimental data. Research in this field is usually supported by a strong experimental infrastructure. Among other factors, quality depends on the accuracy of the measurements. Indeed, many important decisions are based on measurement. For example, the results can be used to assess the conformity of a product or to check a material against a specification/statutory limit. Whenever decision is based on measurement, it is important to have an indication of the quality of the results.
Experimental models (EM) are used for three main tasks:

- The validation of mathematical/numerical models;
- The determination of the necessary input parameters for mathematical/numerical models;
- The calibration of input parameters.

In model validation, experimental models are used to qualitatively and quantitatively compare the results of experiments with simulated output. The result of the experiment often only takes aleatoric uncertainties into account [1, 2, 3, 4, 5]. However, epistemic uncertainties in experiments should also be considered in order to evaluate the accuracy and reliability of computational simulations in engineering design [6, 7, 8]. In many cases, a consistent and quantitative evaluation of the uncertainties of experimental data or the experimental model is not possible. Therefore, in the framework of the research training group, "Assessment of Coupled Experimental and Numerical Partial Models in Structural Engineering (GRK 1462) – Phase II", the German Research Foundation (DFG) proposed a methodology that could quantitatively assess the quality of the experimental results and exemplary implementation.

The simulation of structural behaviour utilises the metrological results of different parameters such as length, mass, and mechanical properties. Other parameters such as geometry, applied load, and environmental conditions are also estimated. Some input parameters of the simulation model are defined based on the metrological results. Such magnitudes will have a certain level of uncertainty. It is well known that an experiment is not perfect. In addition to experimental limitations, internal experiments are commonly affected by environmental factors, form, and experimental technique. In order to obtain a reliable result, these effects must be considered, and the limitations of the measuring system must be respected. In this context, measurement is a complex process that entails physical and mathematical modelling, instrumentation, the processing and validation of data, the evaluation of uncertainty, and the analysis and interpretation of the results; all of these factors contribute accuracy.

Metrology, which is defined as the "science of measurement", mainly focusses on providing measurements of reliability, credibility, universality, and quality. Measurements influence (either directly or indirectly) virtually all decision-making processes. Measurement and metrological operations account for an estimated 3–6% of the gross domestic product (GDP) of industrialised countries [9, 10]. In addition to the identification, quantification, and propagation of metrological uncertainty, the incorporation of metrological aspects such as traceability requirements and the knowledge of operating principles, limitations, and conditions is fundamental for any activity that utilises metrological results. The minimum standards specified for testing machines are inadequate for accurately measuring modulus, and the standard tensile test is not ideal for determining this property [11], as shown in Fig. 1.1. There is quite a large variation in the measured values as well as clear differences between individual bars and EM, Fig. 1.1.

In order to obtain reliable and consistent simulation results, it is necessary to identify, propagate, and quantify the uncertainty of the input parameters in the computational simulation models. This has been the focus of several recent studies [12, 13, 14, 15, 16, 17, 18]. However, there has been no mention of the need to include metrological aspects for characterising the

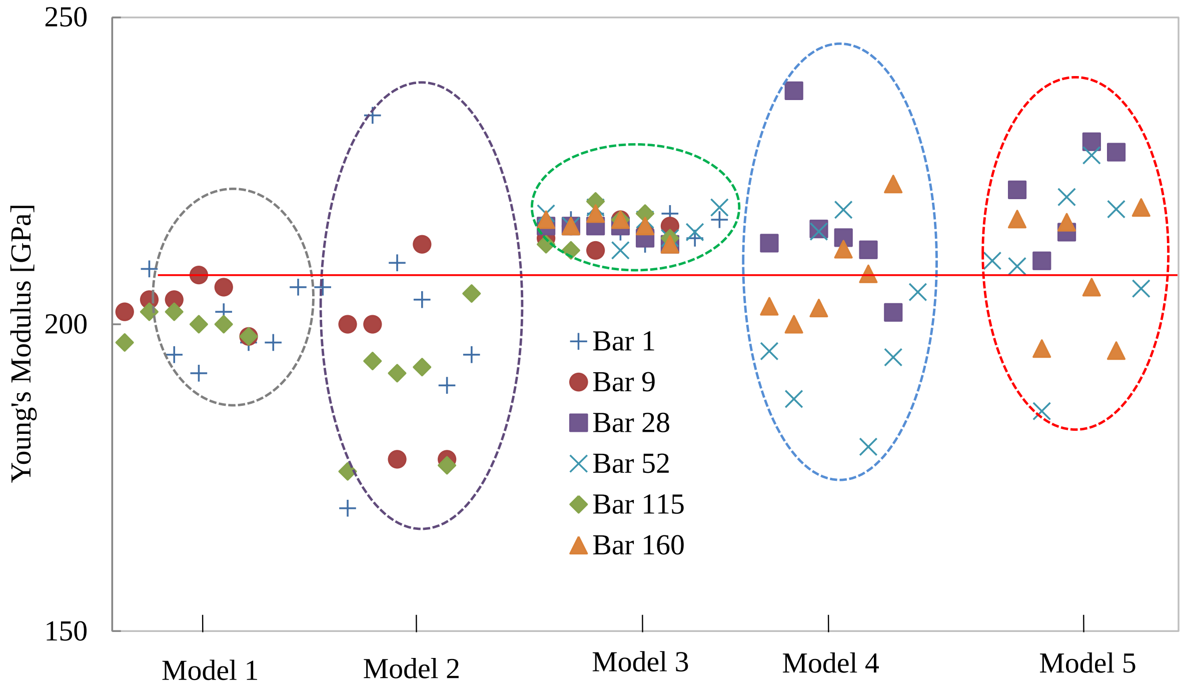


Figure 1.1: Comparison of output in experimental models to measured elastic modulus based on [11]

stochastic behaviour of variables in the model. Within this context, it is therefore crucial to incorporate metrological aspects when determining the uncertainty of the input parameters of computer simulation model and to use techniques that incorporate these uncertainties into the simulation model in order to: (i) ensure the reliability of the results, (ii) correctly evaluate the results of the simulation, and (iii) correctly judge the results in relation to the experimental results, which are often used to validate or verify hypotheses/theories. The result of any measurement is also influenced by uncertainty. According to the 'Guide to the Expression of Uncertainty in Measurement' (GUM) [6, 7, 8], metrological results are simply an approximation or estimate of the value of the measurand. Uncertainty characterises the dispersion of the values that could reasonably be attributed to the measurand. Consequently, the comparison does not concern two mere numerical values but rather a value (limit or threshold) and an interval of values (measure). High values of uncertainty cause wide intervals, thereby increasing the probability of making poor decisions. The result of the comparison strongly depends on the uncertainty of the measurement and can therefore not be disregarded. Underestimating the effects of uncertainty may also lead to poor decisions. There are therefore specific requirements concerning the reliability of measurements. Reliable data cannot be guaranteed simply by selecting the appropriate measurement system. Although the appropriate system should minimise the uncertainty, the measurement must be correctly analysed. In practice, there are many potential sources of metrological uncertainty. These include: (i) the incomplete definition of the measurand, (ii) inadequate knowledge of the effects of environmental conditions, (iii) the metrological method selected, and (iv) the measurement system used. Further information about the metrological process is therefore necessary in order to qualify the reliability of the decision. From this perspective, the appropriate decisional rules are required to compare between metrological data and specifications.

Because of the aforementioned facts, the applicability of the various methods for calculating uncertainty should be reviewed and verified in order to evaluate the assessment of experimental models in the field structural engineering. Despite the obvious need, the evaluation of the quality of the global experimental model has not yet been investigated. A new method is thus needed, especially for assessing the quality of experimental models in the field of civil engineering. This is partly because of the specifications of the models investigated, which require both deterministic and probabilistic analyses. Furthermore, many engineering models cannot be used to simulate physical reality because of the high experimental costs of the method required to do so. The experimental model quality assessment (EMQA) method has been developed with the aim of reducing the uncertainty and cost of experiments as well as increasing reliability and robustness.

1.2 Objectives and Scope of the Study

This thesis is concerned with developing metrological accuracy as well as presenting the procedures used to identify and quantify the influence of the uncertainty of the input parameters on the results generated by computational simulation models. Focus is thus on the development of a methodology for the qualitative and quantitative assessment of the quality of the EM in civil engineering. In this regard, the objectives of the thesis are:

- Specify measurands, mathematically express the relation between measurand and input quantities, identify significant sources of uncertainty, determine input quantities, quantify the standard uncertainties of important single components, identify covariance, and calculate the total uncertainty;

- Review of statistical models and methods of statistical inference that are currently recommended to evaluate measurement uncertainty and determine their applicability to civil engineering research;

- Investigate and implement methods for determining the properties of the model (e.g. sensitivity, robustness, and reliability);

- Develop methods for quality of experimental models based on the total uncertainty and reliability of the predicted output;
- Develop examples that illustrate how to assess the quality of experimental models for the tensile testing of steel, the compressive testing of polymer modified concrete and geotechnical testing (triaxial testing) using the proposed procedures, formats, and models;
- Propose a new concept and verify the adequacy of the existing probabilistic models of sample geometry and mechanical properties that can be used in assessing the probability based reliability of structures;
- Investigate the effects of measurement uncertainty on monitoring data and the monitoring model of concrete poles.

1.3 Limitations and Shortcomings of the Thesis

The present thesis focuses on developing methods for assessing the predicted quality of experimental models. To apply the proposed framework in the evaluation of the quality of experimental models, certain assumptions have to be made. However, these assumptions could lead to the following limitations and shortcomings:

- For Measurement uncertainty, the recommended estimation of error has been adopted in the proposed framework. However, the source of measurement uncertainty can also be classified into various categories. Measurement uncertainty should be further examined and formulated in detail;
- When applying the statistically valid likelihood function, transformed data are still not entirely normally distributed, independent, or correlated. Although robustness, reliability, validity, repeatability, and reproducibility have been investigated, these parameters should be further examined in order to assess the experimental model. The method developed to evaluate quality is verified using a reference model. For this purpose, experimental and monitoring models with defined boundary conditions are used as practical examples. Complex physical models are required for complex structures. However, such investigations have not been included in this thesis because of time limitations and the high cost of the associated studies;
- The scatter of material properties was determined for the interlaboratory comparison. However, even the best-administered interlaboratory testing has practical and experimental limitations. Several complications arose while analysing the tensile test data.

In some tests, the closed loop control mode was not sufficiently optimised, the software was complicated, or the wrong software options were unintentionally used. These factors have all led to unexpected results and errors. Because of missing data, the actual testing speeds cannot be checked. Because of time limitations, the proposed methodology was not applied for synthetic, glass, or composite construction materials.

1.4 Outline of the Thesis

The thesis is organised in seven chapters and two appendices.

Chapter 1 includes a brief introduction pertaining to the background and objectives of the thesis.

Chapter 2 is a literature review pertaining to: (i) evaluation methods for physical models, (ii) the evaluation of experimental models, and (iii) the inter-laboratory evaluation of the quality of experimental data. This chapter discusses the historical background of the analysis of physical model as well as the similitude principles that govern the testing and interpretation of the quality of models.

Chapter 3 presents a stochastic description of the quality of physical models. Stochastic properties such as uncertainty were determined using the GUM, Bayesian, and Monte Carlo methods, local and global sensitivity analyses (SA), and reliability analyses. The robustness, complexity, and cost factor of experiments were also compared between laboratories.

Chapter 4 presents novel methodology for assessing the quality of experimental models. The methodology developed to assess quality is based on total uncertainty, the sensitivity index, the reliability index, experimental design, and experimental costs. The evaluation criteria have a significant influence on the rank of the experimental model quality. This chapter also describes the criteria, statistical approaches, and possibilities for weighting the experimental and monitoring models. Finally, the experimental model quality methods are developed, based on different weighting factors.

Chapter 5 describe the application of experimental models in civil engineering examples. These include (i) the tensile testing of steel, (ii) the compressive testing of polymer-modified concrete, (iii) the triaxial testing of soil, (iv) and the inter-laboratory comparison of experiments. This chapter provides a compressive overview of the relevant statistical characteristics of these parameters with respect to analysing the quality of physical models in civil engineering.

Chapter 6 discusses the application of proposed methods in a model for monitoring concrete poles. This chapter introduces an approach for analysing the quality of strain metrological data, which utilises monitoring data and the associated metrological uncertainties. A new approach for determining metrological uncertainties based on Bayesian updating is also proposed. One example of strain measurement in concrete pole is presented along with the measured reference values of strain measurement.

Chapter 7 presents the conclusions of the present research work and discusses recommendations for future research.

Chapter 2

The State of the Art

This study is concerned with assessing experimental models taking into consideration the different partial models (PM) of experiments. The methods of assessment require an objectives response. Focus has therefore been placed on the assessment and analysis of experimental models. The following sections thus present a brief review of the classification of the model and the concept of general assessment as it pertains to experimental modelling.

2.1 Definition and Evaluation of the Models

2.1.1 Definition of the Models

It is first necessary to design a logical model to analyse a real event. It must then be decided whether the further analysis should be performed using mathematical/numerical or experimental methods. In the first case, the logical model must be described by an advanced "mathematical model" or simplified heuristic mathematical model, which should be based on advanced theoretical perceptions. However, assumptions and suppositions should be investigated using numerical methods such as the Finite Element Method (FEM), Boundary Element Method (BEM), or Finite Difference Method (FDM). However, this only provides an approximation of reality. The reliability of such results is often unknown or difficult to estimate unless it is proven in practice i.e. via measurement. This results in a logical model [19, 20, 21], Fig. 2.1.

Real events can be much more realistically modelled using "physical model", which might be the real objects themselves, prototypes of products and structures, or scaled-down replicas. The ACI Committee 444 [22] defines a structural/physical model as:

Any physical representation of a structure or a portion of a structure. Most commonly, the model will be constructed at a reduced scale.

Generally speaking, physical modelling, especially prototypes, is considered to be costly, time-

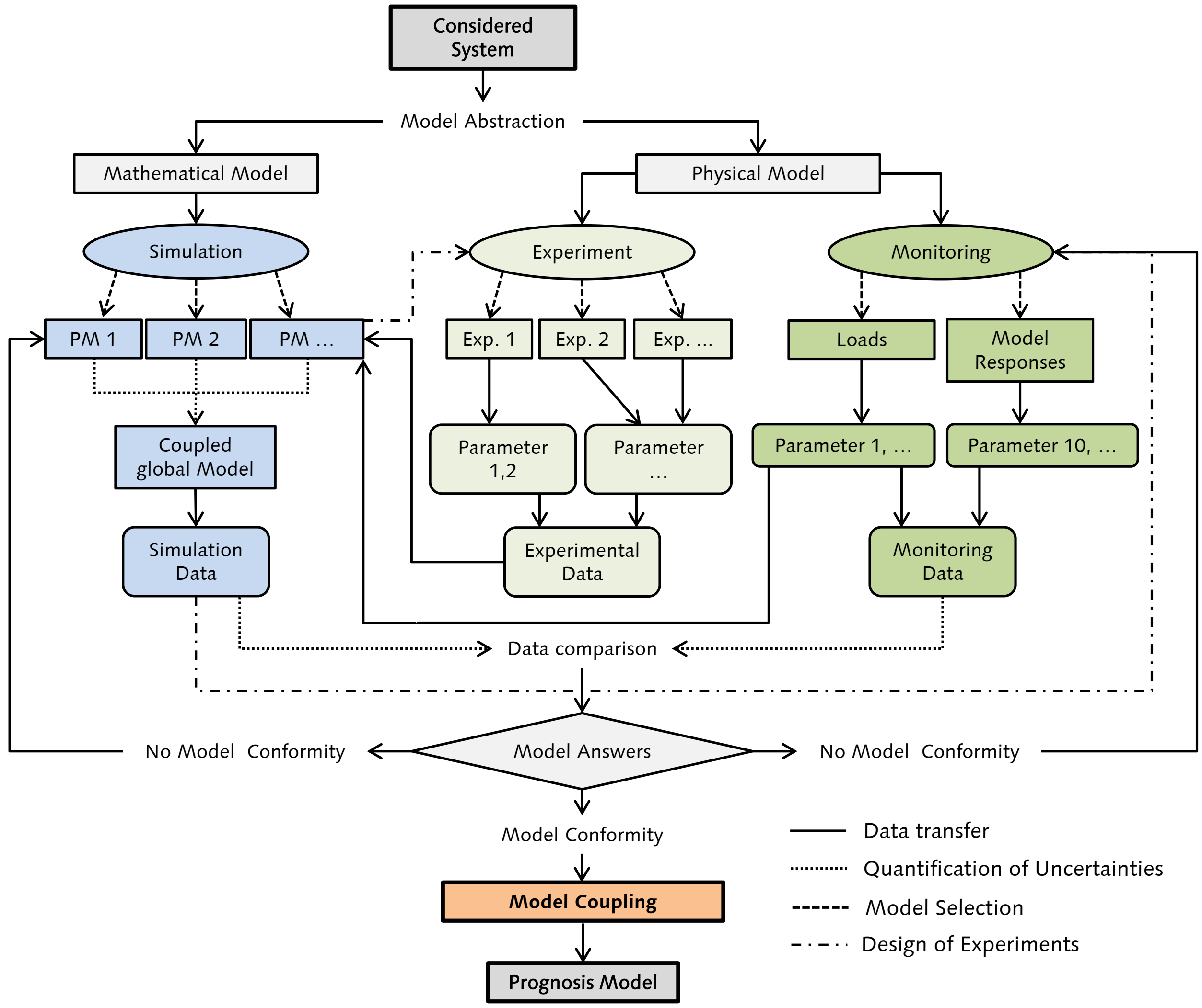


Figure 2.1: Evaluating the quality of modelling

consuming, and sometimes too risky. Physical models therefore require more care than computer-simulation models. In contrast, computer-simulation models should be proven with respect to reality. It is necessary to introduce empirical results and multiple datasets, which are obtained by performing experiments. Experimental investigation entails performing real-world simulations on prototypes. For example, the robust design of experiments or the "Taguchi robustness design" [23] can be utilised to help establish the test and measurement, select the optimal type, number, and location of sensors, and determine the best location for excitation devices for model tests.

When developing experimental methods, it is important to note that while increasing the resolution of equipment and measuring systems can improve the quality of the output, it also rapidly increases the amount of data that must be analysed. A powerful computer is thus required to control the metrological system and the process itself as well as to acquire and evaluate data. In modern experimental analysis, proper theories as well as advanced mathematical models of the experimental methods and the event to be investigated must be introduced. In the context of the research training group, “Assessment of Coupled Experimental and Numerical Partial Models in Structural Engineering (GRK 1462)” the German Research Foundation (DFG) has further investigated and developed methodology to evaluate experimental models and hybrid experimental and mathematical/numerical models. Different steps of combining mathematical/numerical and experimental procedures in analysing mechanical problems are possible as shown in Fig. 2.1. The detailed methodology has been discussed by Scheiber et al. [16]. These steps depend on various conditions such as the research question itself, the purpose and objectives of the investigation, the required accuracy, the availability of experimental equipment, the processing capacity of the computer capacity, and the expertise of the investigator [24].

Experimental investigation is a valuable technique for gaining an increased understanding of material response and structural performance. Indeed, structural testing is particularly indispensable when the numerous uncertainties in material behaviour, structural geometry, and boundary conditions provide a reliable numerical simulation of how structural elements respond [25].

2.1.2 Evaluation Methods for Physical Model

Many statements about the quality of a simulation model can only validated by including the appropriate experiments e.g. the quantification of the statistical uncertainties of model input parameters while calibrating the confidence level of the model [26, 27]. This strongly depends on the definition of the experiment and the quality of its implementation. Research is thus focused on developing methods to quantitatively assess the quality of experimental models and their exemplary implementation in the field of structural engineering. As a result, more engineers will recognise that measurement is relevant in making decisions about the quality of output and services and that information about metrological results are rarely complete. Confidence in measurement is therefore only possible if the measurement uncertainty is quantitatively and reliably assessed.

When comparing model simulations and experiments, the influence of experimental uncertainty is considered. Experimental uncertainty can be expressed in two ways. The first is the uncertainty associated with parameters derived from experimental measurements that are used as model input (model input uncertainty). The second is the uncertainty associated with the experimental measurements themselves (measurement uncertainty). The optimal complexity of the model highly depends on the size and quality of the data (Fig. 2.2). For datasets that are noisy and limited in size, a simple model is needed to prevent the increased prediction error that can result from an overly complex model.

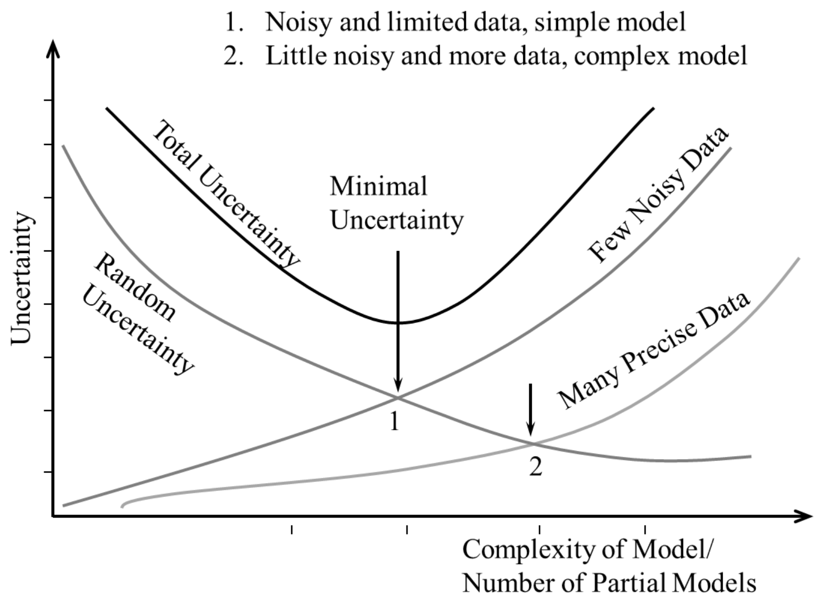


Figure 2.2: Scheme for the size-and quality-dependent uncertainty of prediction, which influences the estimation uncertainty

A method of calculating the overall measurement uncertainty for any type of measurement has only recently been developed. Conventional uncertainty analysis via the root sum square (RSS) method often proves difficult in complex systems and requires approximation at each stage of processing, thereby placing serious doubts on the validity of the results. In accordance with the calibration and testing guide ISO/IEC 17025:1999 [28], all calibration or testing laboratories must apply procedures to evaluate uncertainty in measurement as a guarantee of their technical competence. In order to perform this evaluation, the GUM [6] has been widely used and accepted by the metrological accreditation organisation. Various supplements to the GUM are being developed, which will be progressively put into effect. In the first of these supplements (GUM S1 and S2) [7, 8], an alternative procedure is described for the calculation of uncertainties: the Monte Carlo Method (MCM) and an extension to any number of output quantities. These include non-symmetric measurement uncertainty distributions, non-linearity within the metrological system, input dependency, and systematic bias. The main elements of the formalism were originally proposed by Weise and Elster [29]. Later, the procedure was succinctly outlined by Elster et al. [30] and focused on its relation to the MCM as described by the Joint Committee for Guides in Metrology (JCGM) [7]. Wübbeler et al. [31] explained similarities and differences between the approaches of the GUM and GUM S1. Many recent papers have applied a Bayesian updating to evaluate metrological uncertainty e.g. [29, 32, 33, 34, 35, 36], and several books [37, 38, 39] also discuss issues relevant to this general evaluation method. The approach introduces a state of knowledge distribution about the quantity on interest, which is derived from information about the quantity as well as other influences of quantity and measured data using probabilistic inversion or the inverse evaluation of uncertainty.

Uncertainty characterises the dispersion of the values that could reasonably be attributed to the measurand. Consequently, the comparison does not concern two more numerical values but rather a value (limit or threshold) and an interval of values (measure). High values of uncertainty cause wide intervals. The probability of making a poor decision is thus higher. The result of the comparison strongly depends on the metrological uncertainty and can therefore not be disregarded.

Sensitivity analysis is an important part of metrology, particularly for the evaluation of metrological uncertainties. For example, in experiments, it is interesting to find influence quantities with a high potential for reducing metrological uncertainty. Hence, a measure for the importance of influence quantities or part of an experimental model according to measurement uncertainty is highly relevant. A tool known as local sensitivity analysis has been developed [6]. JCGM [7, 8], which deals with MCM, provides a similar sensitivity index known as 'One at a time' (OAT). Another method is to apply variance analysis that uses properties and capabilities of the MCM e.g. using Sobol' sensitivity indices [40, 41, 42, 43]. Other sensitivity indices have been developed but have not yet been used in metrology.

A further quality measure derived from metrological uncertainty is reliability. In the 1970s and 1980s, heuristic evaluation procedures derived from the field of medicine were used to assess the reliability of metrological data [44]. However, this approach is not applicable for evaluating experimental models in engineering. The reliability of metrological data has been discussed [45, 46], although the fact remains that reliability also depends on the amount of metrological data and calibration of instrument and not just on the metrological uncertainty of the measurement method. With respect to the assessment of quality, the reliability of metrological results refers to the concept of metrological traceability and the associated metrological uncertainty (measurement uncertainty).

The reliability parameter is also known as a hypothesis test for bias e.g. paired t-test, analysis of variance, interclass correlation (ICC), standard error of measurement (SEM), coefficient of variance (CV), repeatability coefficient, or Bland and Altman 95% limits of agreement [47], which are normally used to compare metrological methods. The reliability of experimental models is expressed as a value between zero and one. A value of one corresponds to zero measurement uncertainty, and a value of zero corresponds to zero meaning. As a dimensionless quantity, it is arguably quite difficult to interpret and decide which value is sufficiently reliable for assessing quality [48].

Robustness methodology is intended as a cost-effective approach for improving the quality of experiments, products, and system [23]. Designing robust parameters involves choosing the optimal level of the controllable factor in order to obtain a target or optimal response with minimum variation. Robustness entails adjusting the levels of control factors so that the variability is reduced and the reliability of the measurement is increased.

If the amount of data increases, the standard uncertainty of an experimental model decreases. However, the cumulative uncertainty of the data will increase. Hence, the final uncertainty of the measurement, which is the amount of standard uncertainty and systematic and random deviation, does not monotonically decrease. Consequently, for each dataset, the optimal complexity of the model must be found, whereby the complexity of the models is directly related with the number of variables utilized by the model, which is shown in Fig. 2.2. In the field of experimentation, it is extremely difficult to determine the optimal models.

A popular strategy in structural engineering is to combine several statistical terms that cover different aspects of experimental models. The combined terms have been shown to outperform any single term. EMQA programmes are used to assess models generated by various methods, and the quality of the models range from coarse models, which often have the incorrect magnitude, to highly-accurate models [14, 15, 16]. Assessment functions that consist of several model properties and that are optimised on a diverse set of models will therefore be more suitable for the task of discriminating between good and bad models.

2.2 Physical Models in Structural Engineering

2.2.1 Introduction

In the early to mid 20^{th} century, engineering research and education relied heavily on the use of physical models and experiments to reinforce various topics [49, 50]. Physical models (and reduced-scale structures sometimes called replica models) have always played a significant role in research, design, and education. Experiments on reduced-scale structures and specimens have always been important in civil engineering research. Various problems are encountered in the planning, implementation, and interpretation of experimental studies of structural behaviour. Each of these areas ranges from theoretical similitude requirements to the rather extensive discipline of experimental stress analysis. Small-scale physical models have been used to determine the most efficient form of pure compression and tensile structures since the 17^{th} century and to predict the structural behaviour of some full-sized structures since the mid-19^{th} century [51].

2.2.2 Physical Models – Definitions and Classification

The term “physical model” invokes different images for different people, depending on personal experience. Providing a precise and all-encompassing definition of physical modelling therefore becomes a difficult task. To some, the term physical model might be limited to small-scale reproduction of prototypical situations of structures. One definition of physical model as it applies to structural engineering is discussed in section 2.1.1. A variety of structures may be modelled, including bridges, dams, and towers as well as structures designed for aerospace and underwater. A structural model can also be defined as *any structural element or assembly of structural elements built to a reduced scale (in comparison with full-sized structures) which is to be tested, and for which laws of similitude must be employed to interpret test results* [52]. Applied forces may represent static, seismic, thermal, and aeolian loads. Depending on the function of a model, it can be classified into one of several widely accepted groups. The following discussion gives an overview of some of the most popular classes of models but does not necessary include all classes [51].

As the name indicates, *elastic models:* are specifically used to test behaviour in the elastic range of the material. The geometry of the model and prototype are similar. The material does not necessarily resemble the prototype, but it should be homogeneous. Elastic models using a low modulus of elastic materials are particularly useful for illustrating structural behaviour.

Plastic models: in the case of plastic models, a structure can be designed to form a preselected yield mechanism at ultimate load level leading to a known and predetermined response during extreme events. This has special significance in the context of performance-based design philosophy, whereby it is essential for the structure to deform in a preselected manner in order to achieve the desired levels of performance. Plasticity ends in the failure of material.

Indirect Models: are used to determine the influence diagrams for reaction and internal shear, bending, and axial force. Loads applied to the model do not correspondence to the actual loads expected in real life. Furthermore, there is no need for physical resemblance between the model and prototype. However, if the behaviour is controlled by the quantity, properties such as flexural stiffness should correspond.

Direct Models: are similar to the prototype in terms of both load and symmetry. Therefore, for the prototype, strain and other measurable quantities will be representative for the given load application. Elastic models can also be direct models.

Strength models:, which are also referred to as ultimate strength or realistic models, are direct models made of materials similar to the prototype materials. These models will thus predict prototype behaviour for all loads up to failure. Modelling the behaviour up to the point of failure is clearly an advantage, but finding the proper materials and fabrication techniques for the models can be problematic.

Wind Effect Models: are further classified into shape/rigid and aero-elastic categories. The shape and rigid models are used to measure the forces or wind pressures at various points on the structure. Aero-elastic models, on the other hand, use shape and stiffness properties of the prototype so that wind-induced stresses, deformations, and dynamic interaction can be measured using the model.

Dynamic Models: are used to study vibration or dynamic loading effects on structures. One popular method for seismic simulation is the use of shake tables. Dynamic models can also be used to study internal or external blast effects as well as the effects of impact on structures.

2.2.3 Physical Models History

Since the late 17^{th} century, hanging models (and, since the 1950s, other tensile models) have been used to establish a variable structural form or demonstrate that a certain form is variable. Since the 1930s, scale models have been used to predict stresses, moments, deformations, collapse loads, and safety factors for full-sized structures – especially thin-shelled, reinforced concrete structures – as a means of corroborating predicted quantities using structural calculations.

Small-scale models date back hundreds and even thousands of years. The earliest models were much different from current ones; they were used primarily for planning and constructing [22, 51]. In fact, they can be compared to current architectural models. These early models are not comparable to current structural models because strain, displacement, and force had not been measurable quantities at that time. Some of the earliest examples of structural modelling include the Hoover Dam, which was built in 1930, as well as the other large dams of that era, which were built by the Bureau of Reclamation, Denver, CO [51].

Harris and Sabnis [51] explain the comprehensive overview of structural modelling and list the following specific examples, which are suitable for structural modelling studies in the design phase.

2.2.4 Physical Modelling Process

A physical model must be created by a physical modelling. The detailed planning of an experiment is even more essential than planning an analytical approach because it is usually impossible to refine a physical model halfway through the modelling process. A typical physical modelling process can be broken into the following steps:

- Definition of the scope of the problem i.e. deciding what is and is not required from the model;
- Specification of the similitude requirements for geometry, materials, or loading;
- Decision on the size of the model as well as the required level of reliability or accuracy;
- Selection of materials;
- Planning of the fabrication;
- Selection of instruments and recording equipment;
- Observation of the response of the model during loading;
- Analysis of the data.

Physical modelling includes simplifications concerning the structural properties and actions adopted in the behavioural model. The resulting physical model will be subjected to structural analysis using experimental methods.

2.2.5 Experimental Activity: Measurement and Traceability

The experimental activity has a relevant role in the structural engineering research, being used for modelling physical conditions and to test solutions in order to validate scientific approaches. The measurement of quantities plays a relevant and central role in this activity, where it is expected that they are quantified in a rigorous way, obeying to the required accuracy and being traceable to International System of Units (SI) standards defined by the standards of the GUM [6, 7, 8, 28].

Considering the correctness of the functional relations applied, the critical steps to measurement quality are strongly related with the following tasks:

- *Identification of the measurands*: this task is relevant because it provides an interface between mathematical modelling and physical modelling; in some cases, because of the complex nature of some quantities, multi-stage indirect measurements must be provided;
- *Selection of scientific instrumentation with the required level of accuracy*: this task can be critical when the measurement requirements are near instrumentation resolution, when there is a random dynamic behaviour of the system or when influence quantities significantly affect the measurements;

- *Calibration of instrumentation*: in some cases, calibration could be a problem because of the difficulty to define reference conditions or standards in order to provide the connection to the SI traceability framework;
- *Calculation of correction curves*: the development of mathematical algorithms in order to correct calibration deviations can be complex when dealing with multivariable systems;
- *Evaluation of measurement uncertainty*: the measurement process can occur in different frameworks, being obtained under static or dynamic conditions, being described on complex scalar or matrix basis, having non-linear functional relations, among others, which may be not compatible with the requirements of conventional methods used to evaluate measurement uncertainty [6, 7, 8, 28].

2.2.6 Advantages and Limitations of Physical Model Analysis

The main advantage of physical models over analytical models is their behaviour can be completely characterized to the point of failure. Although great advances have been made in computer analysis programs, it remains difficult, if not impossible, to predict the failure capacity of three-dimensional structures under complex loads.

Cost is an ever-relevant issue for modelling and experimental studies. Scaled models provide savings on materials, labour, sensors, preparation, disposal, facilities, and laboratory equipment. A good example of cost savings is demonstrated by the fact that a concentrated load is reduced in proportion to the square of the geometric scale factor of the model.

However, despite the cost savings and behavioural advantages of using physical models, they cannot be particularly well applied to typical design environments. Analytical models are typically less expensive and faster. Physical models are therefore more suitable for cases in which analytical models are neither adequate nor feasible. Research institutes and facilities commonly implement physical models. For this thesis, the most relevant research application is the development of experimental data to verify the adequacy of the proposed analytical methods. This will be expanded upon in later sections.

2.3 Quality Assessment of Experimental Models

In sections 2.1.1 and 2.1.2, it was stated that uncertainty can be regarded as a quantitative indication of the quality of the measurement. In general, measurement uncertainty comprises many components. Some of these components may be evaluated from the statistical distribution of the results of a series of measurements and can be characterised by experimental standard deviations. The other components, which can also be characterized by standard deviations, are evaluated from assumed probability distributions based on experience or other information [53]. It is clear that the result of the measurement is the best estimate of the value of the measurand and that all components of uncertainty, including those arising from systematic effects such as components associated with corrections and reference standards, contribute to dispersion [6, 7, 8, 28]. Uncertainty can be estimated in one of two ways:

- Type A uncertainty estimates are obtained by the statistical analysis of data – for example, repeatability may be estimated as the standard deviation of a set of replicates;

- Type B uncertainty estimates are obtained by other means, such as finding the uncertainty of a calibration result on a calibration certificate or the uncertainty in the value of a reference material from the material certification. In some cases, uncertainty estimates can be based on one's knowledge and experience, on the laws of physics or from knowledge about how an instrument behaves.

Measurement uncertainty can be evaluated within the framework of the conventional theory of statistics [54] e.g. least squares, maximum likelihood, and bootstrap as well as Bayesian statistical theory. Both types of statistics take considerably different approaches to the concept of probability. In statistics, the conventional concept of probability is associated with the relative frequency of random events. In the case of systematic effects, non-linear measurement models, and values measured close to detection limits, such statistics fail [55, 56]. An example of inconsistencies occurring in conventional statistics can also be the guide, which has introduced two different methods of evaluating uncertainty (Type A and Type B) [6]. Similarly, the Bayesian approach and MCM treat random and systematic effects in the same way. In this thesis, the three aforementioned methods for determining uncertainty will be implemented using examples from civil engineering. The experimental models are classified as the intra-laboratory approach (based on a distinction between the evaluation of uncertainty carried out by the laboratory itself) and the inter-laboratory approach (uncertainty evaluation based on collaborative studies). These approaches have been presented in the guidelines on the expression of uncertainty in quantitative testing [57]. Finally, the quality of different experimental models is defined based on the measurement uncertainty.

In such a context, the uncertainty budget associated with the evaluation of measurement uncertainty is useful for gaining a better knowledge of measurement process through the contributions to the variance of output quantity of each input quantity. Within the framework of the GUM, the partial derivatives of the measurement mode that are computed are the best estimate of each input quantity. This concept of sensitivity analysis is used in the GUM. In contrast, the global sensitivity analysis is used in the Bayesian and MCM framework [7, 8].

On the other hand, the reliability of the experimental model is inversely proportional to the measurement uncertainty. If reliability is high, measurement uncertainty is small. The CV and reliability index (β) are used for quantifying the quality of the experimental model.

The experimental model used to perform the experimental tests need to certify the traceability as well as estimate the levels of measurement uncertainty. Experimental activities are important and necessary for comparing results between experimental models designed for scientific and technological purposes. Thus, the results obtained by different experimental models can only be used objectively if the measurement results are traceable and the experimental models have assessed the respective measurement uncertainty and conformity assessment. Conformity can be made on a quantitative basis- referred in statistical acceptance sampling by variable.

2.4 Quality Assessment of Monitoring Models

For quality assurance of measurement results, the evaluation of measurement performance is especially important because measurement results depend on both the measurement method and measurement model. Mathematical/numerical model parameters are usually estimated through minimization algorithms with respect to experimental data. However, the values obtained in the classical minimisation approach are not always correct and require critical evaluation where the minimum of the cost function is attained [58]. Monitoring is a key factor in the reliable and safe design of structure, while the term monitoring results involve some level of uncertainty may originate from causes such as the lack of accuracy in measurement equipment, random variation in the measurands, approximations in data reduction relations, observation, and supervision of an activity or a process [59, 60, 61]. Those uncertainties can be caused by either systematic or random deviations in the measurement. All of these individual deviations influence the result. For this purpose, a typical example of a monitoring model for uncertainties quantification was used. These will be discussed in chapter 6. The measurement uncertainty based on measurement process, observation equation, and the Bayesian method was determined. The approach introduced in this section utilizes both types (systematic and random) for the derivation of a posterior measurement uncertainty by Bayesian updating [35]. This facilitates the quantification of a measurement uncertainty using all available data of the measurement process. The measurement uncertainty models derived in this monitoring model are analysed with a sensitivity study and discussed in detail resulting in the identification of the most relevant sources of measurement uncertainty.

From this application example, an engineer can easily understand how the reliability of the estimated parameter increases with increasing quality of data and decreasing data measurement uncertainty. The quality of monitoring models is evaluated based on the metrological uncertainties. Measurement uncertainty arises from calculating the covariance matrix for the measurement uncertainty. This is because increasing the number of independent measurement devices does not steadily decrease the measurement uncertainty as shown in Fig. 2.2. The analysis of uncertainty is a vital part of any monitoring models or experimental programme.

Chapter 3

Stochastic Description of Physical Model Quality

3.1 Introduction

Civil engineering encompasses a wide and diverse range of domains, each one dealing with specific issues in which the experimental and monitoring components often play an important role. In recent years, rapid changes have been made in materials technology. Worldwide, substantial investment will continue to be made in the development, processing, and application of materials. The use of modern materials pervades all of industry and strongly influences competitiveness. However, to convert these materials into competitive engineering products, the designer requires access to appropriate design methodologies that specify material property requirements. The need to generate reliable design data therefore becomes crucial. Furthermore, the quantitative assessment of the behaviour and performance of materials is essential to the quality and reliability of products. The measurements of materials have an important and widespread influence [62, 63, 64]. This chapter presents studies on the stochastic description of the quality of physical model, taking into consideration different physical properties.

3.1.1 Metrology Contribution to Measurement Quality

It is widely accepted today that metrology is one of the pillar that hold quality. Measurement accuracy achieved, by this way, a relevant role in the quality. The quality of an experiment may need to be assessed for various reasons, one of which is finding is the best combination of inputs for experiments. Another is to ensure metrological quality. A metrological–probabilistic approach can provide valuable mathematical and computational tools that are especially well suited to the study, evaluation, and improvement of measurement various components (e.g.

modelling, instrumentation performance, data processing, data validation, and traceability) of metrological processes. Emphasis is placed on evaluating the quality of measurements in order to analyse results and promote the quality and capacity associated with decision-making [65]. The evaluation of quality evaluation based on analyses of uncertainty, sensitivity, and reliability are important because they can be generically applied. Such procedures are employed to quantify the uncertainty, propagation of uncertainty, rank of input parameters and reliability of data in order to identify the main contributors to the uncertainties of the measurement models quality evaluation as well as to identify priorities for further investigation and use.

3.1.2 Characterization of Errors and Uncertainties

Measurement error is described as the difference between a measurement and the true value of the measurand (the quantity being measured) [66]. Error does not include mistake. In turn, a true value is defined as a value that is consistent with the definition of the measurand. The uncertainty of a measurement is the range of the measured value in which the true value or the conventional true value of the measured quantity is likely to lie at the stated level of confidence [67]. The characterisation of uncertainty is not trivial; it entails developing methods to model both random and systematic uncertainty [68, 69]. Regardless of the type being considered, the characterisation of uncertainty depends on data to provide the insight needed to define a stochastic model of the appropriate behaviour of the measurement/simulation of the parameter [70].

3.2 Uncertainty Analysis

The uncertainty of direct physical measurements is a well-accepted concept in engineering. In simple terms, it is a representation of the likely values of the metrological results. The concept of uncertainty is sometimes confused with the concept of error. Error refers to the difference between the measurand (i.e. the true value of the quantity to be measured) and the metrological result. A measurement can be close to the unknown value of the measurand, thus yielding negligible error; however, it may have a large uncertainty. Because the exact value of a measurand can never be evaluated, error is an abstract concept and can never be quantified. However, uncertainty is a measure that can – and should – be quantified for every measurement. It is not a mere repeatability, but it is at least as high as the intra-laboratory reproducibility. If it is an attribute of a general analytical test, it is at least as high as the inter-laboratory reproducibility. Measurement uncertainty can be determined by the addition of the variances of the individual step of the test procedure or by an approach which starts with one of the above-mentioned reproducibilities. Any measurement uncertainty should be kept low but it is objectionable to state to low a value, e.g. by falsely reporting mere repeatability data instead of properly determined uncertainty data.

The probabilistic presentation of uncertainties in direct measurements is formalized by International Organization for Standardization (ISO, GUM) and United States National Institute of Standards and Technology (NIST) [71, 72]. The GUM provides generals guides for the evaluation and expression of measurement uncertainty for a range of measurement process [67]. The measured value (MV) expresses exactly what we can determine safely on the value of the measurand, based on the application of the measurement system (SM). It consists of two parts:

- The base result (BR), which corresponds to the central value of the range which shall be within the true value of the measured;
- The measurement uncertainty (MU), which expresses the range of doubt still present in MV caused by errors contained in the SM and/or variations of the measurand, and should always be accompanied by measuring unit. Thus, the MV must always be expressed as:

$$MV = BR \pm MU[unit]. \tag{3.1}$$

When analysing uncertainty, the measurand should be clearly specified. Although this may seem trivial, it is nevertheless essential [73, 74]. Without clearly understanding the purpose of the measurement and the factors that influence the results, it is impossible to reliably estimate the measurement uncertainty. The evaluation of the measurement uncertainty is therefore not purely a mathematical task; it depends on the measuring system and environment as well as other metrological factors such as critical thinking and skill.

According to the recommendations of ISO and the NIST, measurement uncertainties should be expressed explicitly when presenting the results of measurements using probabilistic concepts. This is the first step towards implementing the probabilistic approach to solve problems, whereby the direct physical measurements are one of the inputs to the problem. The particular statistical paradigms under which different methods for uncertainty assessment are described include the framework of the GUM as well as the Bayesian and Monte Carlo method, which are discussed in further sections.

3.2.1 The Guide to the Expression of Uncertainty in Measurement (GUM)

The most widely used and accepted method for evaluating uncertainty evaluation is the GUM [6, 7]. The GUM characterises quantities using either a normal (Gaussian) or t-distribution, which allows measurement uncertainty to be delimited by means of a coverage interval. The GUM approach has two main drawbacks. First, linearisation of the GUM model can lead to an inadequate representation of a system. Second, the probability density function (PDF) for the output quantity can appreciably deviate from a Gaussian or t-distribution. In the first case, the estimate of the output quantity and the associated standard uncertainty provided by the GUM might be unreliable. In the second case, unrealistic coverage intervals can be created.

3.2.1.1 Sources of Measurement Uncertainty

According to the GUM, in practice, there are many possible sources of uncertainty in a measurement, including:

- The incomplete definition of the measurand;
- An imperfectly defined measurand;
- A non-representative sample (measured sample size may not represent the measurand);
- An inadequate knowledge of the effects of environmental conditions on the measurement or imperfect measurement of environmental conditions;
- Personal bias error in reading analogue instruments;
- The resolution of the instrument or discrimination threshold;
- Inaccuracies of measurement standard and reference material values;
- Inexact values of constants and other parameters obtained from external sources and used in data reduction algorithm;
- Approaches incorporated into the method and measurement assumptions;
- Identical variation in repeated observations of the measurand under apparently identical conditions.

Fig. 3.1 graphically presents the main sources of uncertainty acting in a measurement process.

3.2.1.2 Modelling of Measurement Uncertainty

For evaluating the measurement uncertainty, it is necessary to mathematically describe both the measurement processes as well as all quantities and parameters influencing the measuring

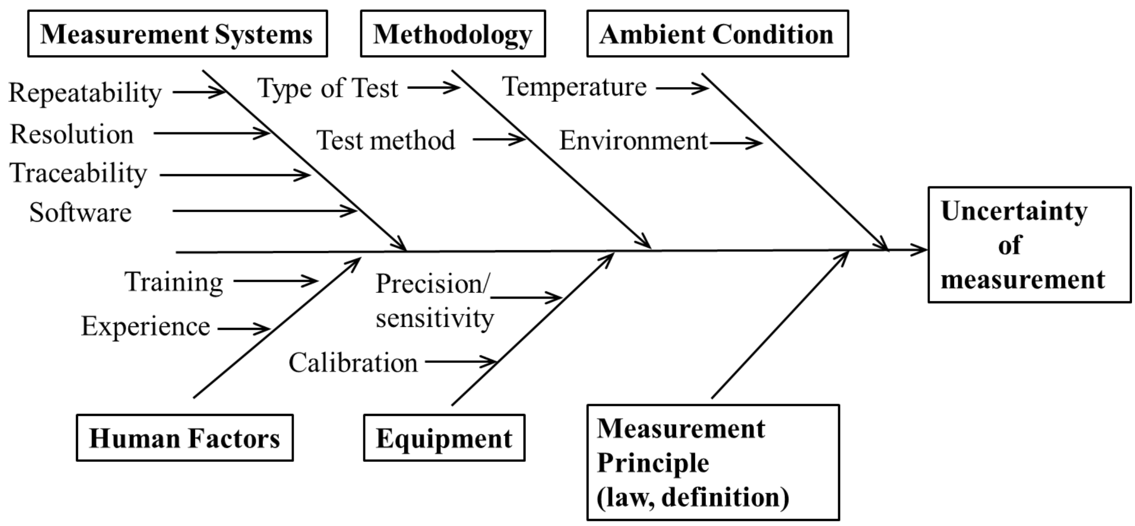


Figure 3.1: Sources of uncertainty acting on the measurement

system [6, 7, 8]. If all quantities influencing the result of measurement can be varied, the measurement uncertainty can be evaluated by statistical treatment of experimental data. However, this is rarely possible in practice because of the time and financial costs involved in the exhaustive task of experimentally evaluating the uncertainty. There are several ways of quantifying the different types of measurement uncertainty [6, 7, 75] as is shown in Fig. 3.2 and Fig. 3.3. In the GUM, a measurement system is modelled, the input quantities, N, say, in number, are denoted by $X = (X_1, X_2, ...X_N)^T$ and the output quantity by Y. The measurement model:

$$Y = f(X) = f(X_1, X_2, ...X_N). \tag{3.2}$$

In most cases, $f(X_1, X_2, ...X_N)$ will be an analytical expression, but there may be cases in which it is described by a group of expressions that include correction factors for systematic effects. This leads to a more complex equation that can either be determined experimentally or that exists only as a computation algorithm, which has to be evaluated numerically, or a combination of the above cases. An estimate of the output Y parameter, denoted as y, is obtained from Eq. 3.2 using input estimates $x_1, x_2, ...x_N$ for the values of N input quantities $X_1, X_2, ...X_N$. Thus, the output estimate y, which is the result of the measurement, is given by:

$$y = f(X) = f(x_1, x_2, ...x_N). \tag{3.3}$$

The estimated standard deviation associated with the estimate of output or outcome y simulation called combined and designated uncertainty $\mu(y)$ is determined by the estimated standard deviation associated with each input estimate x_i and designated standard uncertainty $\mu(x_i)$. Each input estimate x_i and associated standard uncertainty $\mu(x_i)$ is obtained from a distribution of possible values of the input variable X_i. This distribution of likelihood can be

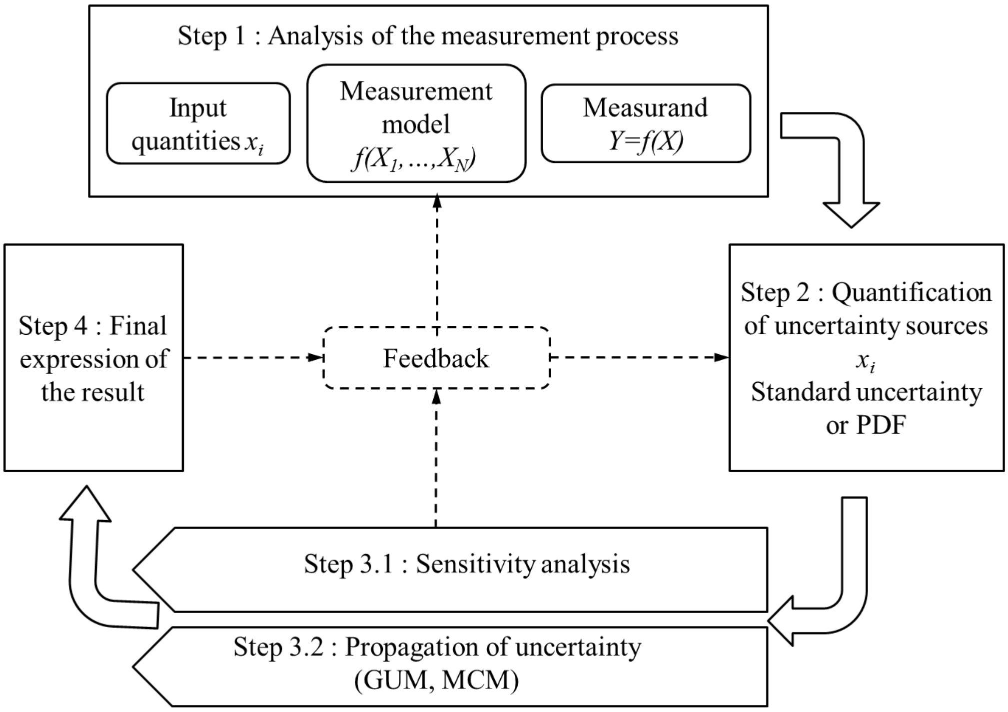


Figure 3.2: The general method to evaluate measurement uncertainty based on [6, 7]

based on the frequency i.e. in a series of observations $X_{i,k}$, of X_i or may be a scientific judgment based distribution, using all relevant information that is available [6, 7]. Fig. 3.3 summaries the ISO GUM procedure.

3.2.1.3 Quantifying the Contribution of the Sources of Uncertainty

The measurement uncertainty associated with the input estimates is evaluated according to the evaluation methods of Type A or Type B [6, 7, 72]. Type A standard uncertainty is evaluated by statistically analysing a series of experimental observations. In this case, the standard uncertainty is the experimental standard deviation that is obtained from a procedure appropriate for calculating the arithmetic mean or regression analysis. Fig. 3.4 shows possible components that cause of uncertainties.

Type A Evaluation of Standard Uncertainty

Type A evaluation of standard uncertainty may be applied when several independent observations for each of the input were made under the same measurement conditions [6, 7, 72]. If there is sufficient resolution of the measuring system, an observable scatter or spread in the values will be obtained.

As an example of a Type A evaluation, consider an input quantity X_i for which the value is estimated from n independent observations $X_{i,k}$ of X_i obtained under the same conditions of

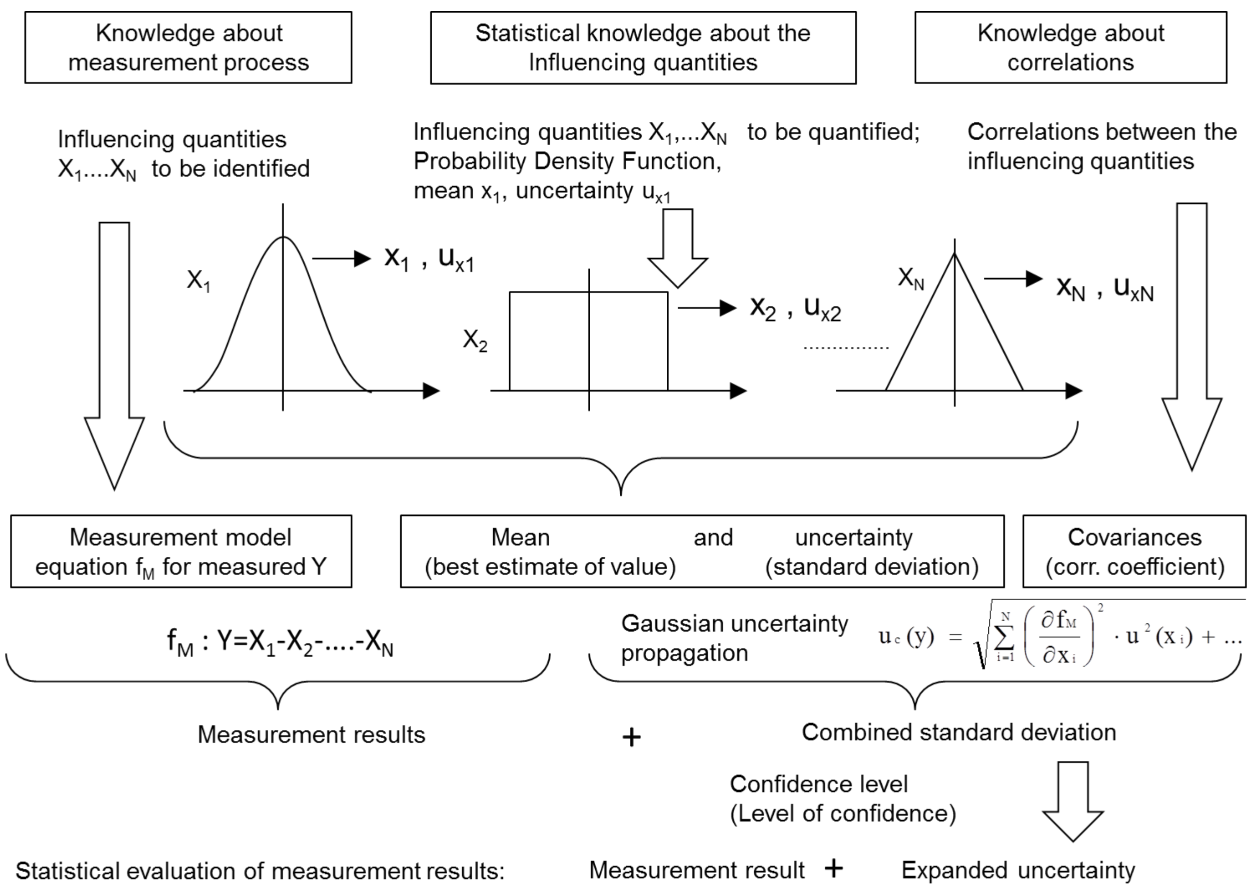


Figure 3.3: Illustration of the concept of GUM procedure [6] and according Sommer and Siebert [76]: The knowledge of the measurement process on quantities that influence the measurement results are quantified, so that a reliable statistical value is finally derived.

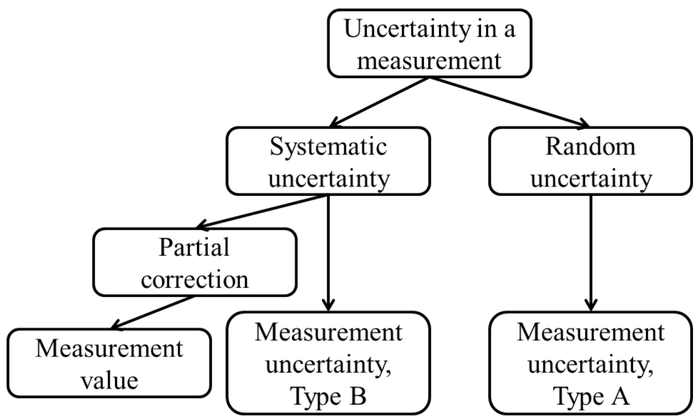


Figure 3.4: Different possible components of uncertainty

measurement. In this case, the input estimate x_i is usually the sample mean:

$$x_i = \bar{X}_i = \frac{1}{n} \sum_{k=1}^{n} X_{i,k} \tag{3.4}$$

and the standard uncertainty $\mu(x_i)$ to be associated with x_i is the estimated standard deviation of the mean:

$$\mu(x_i) = s(\bar{X}_i) = \left(\frac{1}{n(n-1)} \sum_{k=1}^{n} (X_{i,k} - \bar{X}_i) \right)^{1/2}. \tag{3.5}$$

Type B Evaluation of Standard Uncertainty

3.2.1.4 The Law of Propagation of Uncertainties (LPU)

The law of propagation of uncertainty, which has been adopted by many organizations, is used to implement standards and establish guidelines for measuring uncertainties [6, 7, 72]. To apply this law, the values of the input parameters are represented as means and standard deviations of the probability density functions of these variables. The combined standard uncertainty $\mu(y)$ can be determined by using the law of propagation of uncertainty, which is represented by the Eq. 3.6 and Fig. 3.5, is based on a construction of a linear approximation to the model function Eq. 3.2.

$$\mu^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 \mu_s^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(\frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} \right) \mu(x_i, x_j). \tag{3.6}$$

where $\left(\frac{\partial f}{\partial x_i}\right)$ is the sensitivity coefficient, m is the degree of freedom and N is the number of input in measurement model and $\mu(x_i, , x_j)$ are the possible covariances between them.

The LPU in Eq. 3.6, illustrated in Fig. 3.5. The model has mutually independent input quantities $X = (X_1, X_2, X3)^T$ for which the values are estimated by x_i with associated standard uncertainties $\mu(x_i)$, for $i = 1, 2,$. The value of the output quantity Y. is estimated by y, with associated standard uncertainty $\mu(y)$.

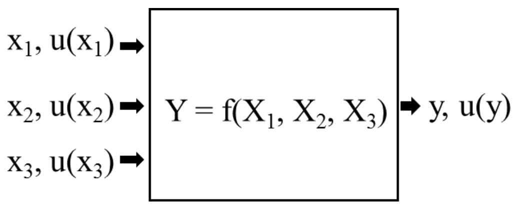


Figure 3.5: Illustration of the propagation of uncertainties for $N = 3$ in put quantities

The sensitivity coefficients describe how the output estimate y varies with change in the values of input estimates $x_1, x_2, ...x_N$. In particular, the change in y produced by a small Δx_i variation in input estimate x_i, is given by Eq. 3.7. For a further discussion of the sensitivity analysis, refer to section 3.4.

$$\Delta y_i = \frac{\partial f}{\partial x_i} \Delta x_i. \tag{3.7}$$

If this change is generated by the standard uncertainty of the estimate x_i, the corresponding change in y is:

$$y = \frac{\partial f}{\partial x_i} x_i. \tag{3.8}$$

The combined variance $\mu^2(y)$ can thus be viewed as the sum of the terms, each of which represents the estimated variance associated with the estimated output y generated by the estimated variance estimate associated with each input x_i. The degree of correlation between x_i and x_j is characterised by the coefficient of correlation, which is defined as follows Eq. 3.9.

$$r(x_i, x_j) = \frac{\mu(x_i, x_j)}{\mu(x_i)\mu(x_j)}. \tag{3.9}$$

where $r(x_i, x_j) = r(x_j, x_i)$ and $-1 \leq r(x_i, x_j) \leq +1$. If the estimates x_i, x_j are independent, $r(x_i, x_j) = 0$. In addition, one variation does not imply an expected variation in the others.

The LPU is commonly written in a matrix format [77], and a compact way of writing the Eq. 3.6, which avoids the use of doubly scripted summations, is:

$$\mu^2(y) = CU_xC^T \tag{3.10}$$

where co-vector of sensitivity coefficient; $C = \frac{\partial y_i}{\partial x_i}$, defined well as the following Eq. 3.11.

$$C^T = \left[\frac{\partial f}{\partial X_1}, ..., \frac{\partial f}{\partial X_N}\right] \mid X_{=x} \tag{3.11}$$

U_x is the uncertainty matrix of order NXN associated with the estimate x of input quantities X.

$$U_x = \begin{bmatrix} \mu(x_1, x_1) & \cdots & \mu(x_1, x_N) \\ \vdots & \ddots & \vdots \\ \mu(x_N, x_1) & \cdots & \mu(x_N, x_N) \end{bmatrix}. \tag{3.12}$$

The diagonal of the covariance matrix is given by the uncertainty of the parameters, which is $\mu(x_i, x_i) = \mu^2(x_i)$. The non-diagonal terms represent the covariance between the parameters of the mathematical model.

The test results were also statistically analysed in order to assess their reliability. The uncertainty was calculated using different references. The uncertainty in measurements of the test parameter is calculated by standard distribution formula.

$$\mu_A = s(\bar{X}_i) = \mu_{rep}. \tag{3.13}$$

Then, combined uncertainty ($U_{combined}$) can be calculated then measurement uncertainty for output quantities as shown below;

$$\mu_{combined} = \sqrt{\mu^2(y) + \mu_{rep}^2}. \tag{3.14}$$

The uncertainty of measurement is expressed as an expanded uncertainty with coverage factor, k (see [6], which is depends upon the degree of freedom):

$$U_E = k\mu_{combined}. \tag{3.15}$$

The GUM provides general guidance for the application of the LPU to the spread of measurement uncertainty. The main stages for the application of measurement uncertainty based on LPU according to GUM is summarized below:

- Define the PDFs of the values of input parameters $X_1, X_2..., X_N$, mean $x = (x_1, x_2, ..., x_N)$, and deviations (standard uncertainty) with uncertainty $\mu(x) = (\mu(x_1), \mu(x_2), ..., \mu(x_N))$;
- For each parameters i, j in which values of X_i and X_j are mutually dependent, define the FDF joint values of X_i and X_j covariance $\mu(x_i, x_j)$ associated with x_i and x_j;
- Determine the first order derivatives with respect to the input parameters;
- Calculate the best-fit value of y by evaluating the model at a fixed value of X equal to x;
- Calculate the sensitivity coefficients of the measurement model at the point x;
- Determine the uncertainty $\mu(y)$ based on the law of propagation of uncertainty;
- Calculate the effective degree of freedom (ν_{eff}) associated with $\mu(y)$ considered for input estimates ν_i by means of the Welch-Satterthwaite equation;

$$\nu_{eff} = \frac{u^4(y)}{\sum_{i=1}^{N} \frac{c_i^4.u^4(x_i)}{\nu_i}}. \tag{3.16}$$

- Calculate the expanded uncertainty U_p and show the confidence interval. Knowing that the probability distribution is a t distribution with ν_{eff} degrees of freedom, the coverage factor k_y can be determined for the desired level of confidence.

Fig. 3.6 depicts the procedure for estimating measurement uncertainty according to the GUM and MCM.

This GUM approach is mainly concerned with univariate measurement models i.e. models with a single scalar output quantity. There may be models with more than one output quantity. However, multivariate measurement models (i.e. namely those with any number of output quantities) can be problematic. Such quantities are generally mutually correlated because they depend on common input quantities. A generalisation of the GUM uncertainty framework [8] is used to provide estimates of the output quantities, the standard uncertainties associated with the estimates, and covariances associated with pairs of estimates. The input or output quantities in the measurement model may be real or complex, which is discussed in Annex A.

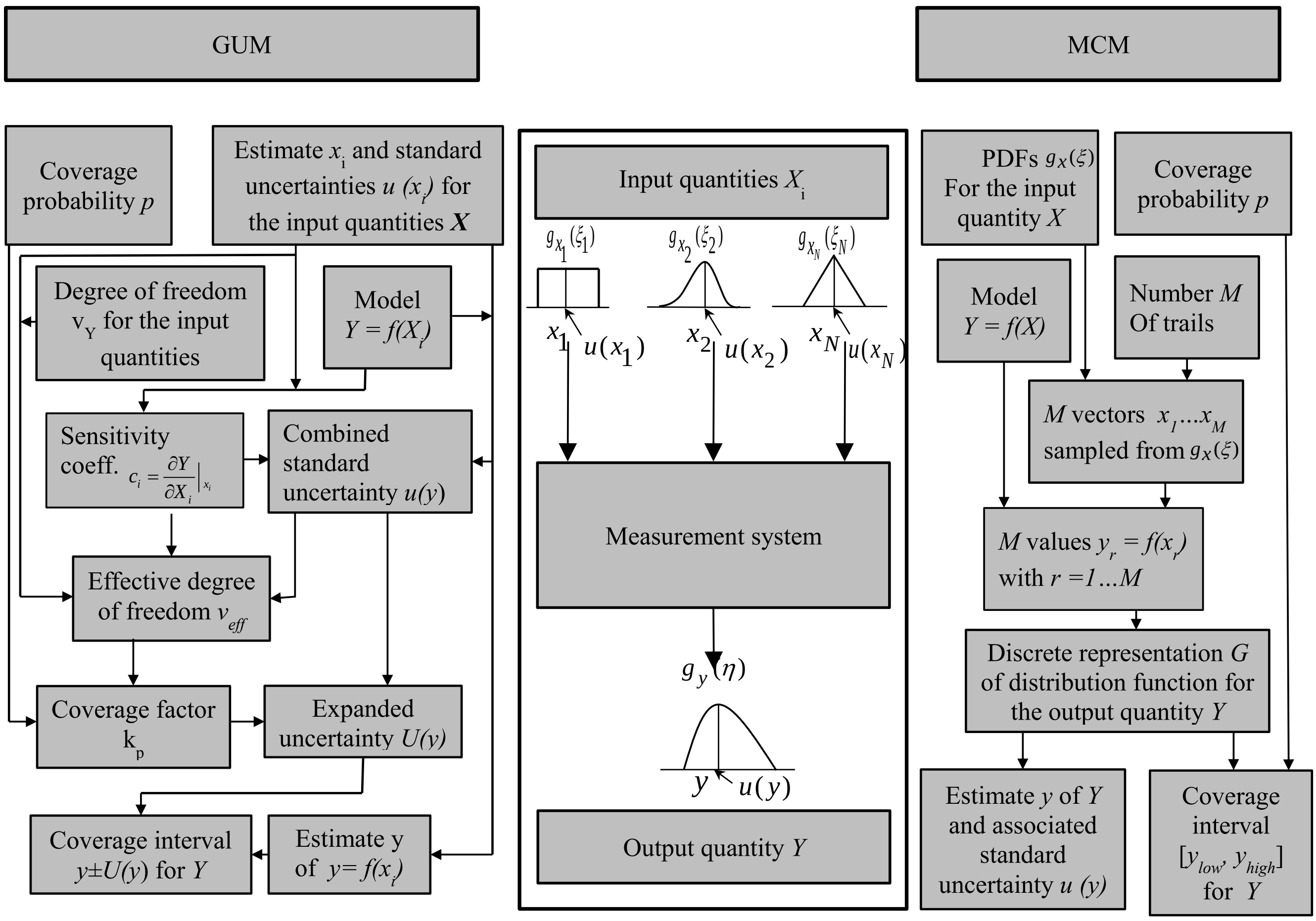


Figure 3.6: Schematic description of the GUM and MCM uncertainty evaluation processes

3.2.1.5 Representation of Uncertainty

In practical application, the values of errors are always unknown to some extent. Therefore, uncertainty is used to describe the possible outcome of an error. Uncertainty can be based on conventional (recommended by the GUM), Bayesian or imprecise probabilities.

The conventional perspective deduces probabilities from a series of repeated random events. For instance, if a random event has a probability of 95%. The conventional approach is hard to validate for civil engineering applications in which usually only one trial is available. Therefore, interpreting probability is not well suited to represent deterministic events in which the uncertainty is associated with measurement uncertainty. Indeed, conventional probabilities are expressed as PDF. Fig. 3.7 depicts a pdf for a random variable X, whereby confidence interval bounded by X_{low} and X_{high} contains a probability $\in [0, 1]$.

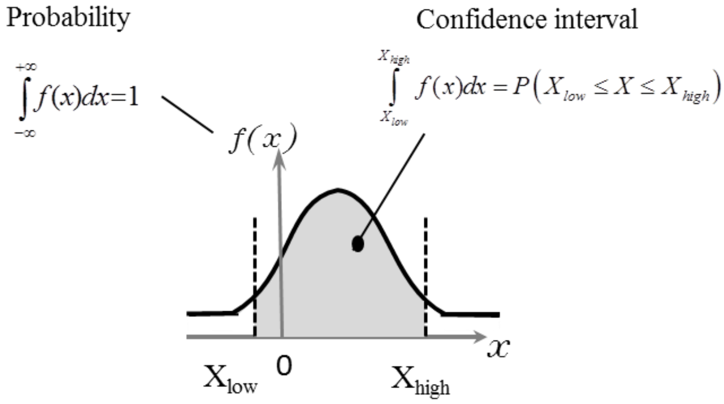


Figure 3.7: PDF of a random variable X, where a confidence interval bounded by X_{low} and X_{high} contains a probability $\in [0, 1]$

3.2.2 Bayesian Method

Bayesian statistics is based on Bayes' theorem. This framework uses a definition of probability that allows probability distributions to be defined without physical data e.g. using manufacturers specifications or other expert knowledge. In most measurement applications, however, physical measurements (data) can be used to estimate one or more of the input quantities. In such cases, the corresponding probability density can be obtained via Bayes' theorem as follows [78]. Probability is understood as a degree of belief or of plausibility of a proposition, which is conditional on all relevant information available about that proposition [29, 32, 33, 34, 35, 36, 79, 80].

Given two propositions A and B, the probability of both of being true is equal to the product of the probability of A being true times the probability of B being true given that A is true, as is shown in Eq. 3.17.

$$P(AB) = P(A)P(B \mid A), \tag{3.17}$$

where the vertical line stands for 'conditional upon' or 'given' for short. However, because A and B can be interchanged, Bayes' theorem follows:

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)} \tag{3.18}$$

where B is the observed data of measuring X.
A is the information I, the value of X lies within a given infinitesimal interval $(\xi, \xi + d\xi)$. A quantity such as X will not be considered a random variable but rather as an entity with which an information-based probability density function (ipdf), $g_x(\xi \mid I)$ can be associated based on the given information I. The domain of this function extends over all possible values ξ of X

and can thus be either discrete or continuous. The ipdf encodes the state of knowledge about the quantity, such that $g_x(\xi \mid I)d\xi$ is the information-based probability that the value of X lies in the infinitesimal interval $(\xi, \xi + d\xi)$ [7].

Therefore, $P(A) = g_x(\xi \mid I)d\xi$ and $P(A \mid B) = g_x(\xi \mid d, I)d\xi$, where $g_x(\xi \mid I)$ and $g_x(\xi \mid d, I)$ are information-based probability density functions that describe the states of knowledge about X without and with data, respectively. The data may consist of several pieces of information, one of which might be the series of measured values $x = (x_1, ..., x_n)$. The factor $P(B \mid A)$ reflects the belief about obtaining the data given that the hypothesis $X \in (\xi, \xi + d\xi)$ is true. This is called the likelihood of the data and is written as $L(\xi; d)$. Bayes' theorem then becomes

$$g_x(\xi \mid d, I) = CL[\xi; d)g_x(\xi \mid I) \tag{3.19}$$

where C is a normalization constant, such that the integral of $g_x(\xi \mid d, I)$ over all possible values ξ is one.

The likelihood reflects the way our beliefs about having obtained the given data vary with the possible values of the quantity. It is not an ipdf, so it does not need to be normalized. In fact, the likelihood can be multiplied by an arbitrary constant, (or in general by a function of the data alone), because the variable is ξ and the multiplier can then be absorbed into the constant C.

If no data are available, the likelihood becomes a constant and $g_x(\xi \mid d, I)$ becomes equal to $g_x(\xi \mid I)$. Also, if $g_x(\xi \mid I)$ can be taken to be a constant, and if $L(\xi; d)$is symmetric, the expectation of $g_x(\xi \mid d, I)$ is equal to the corresponding maximum likelihood estimate in conventional statistics. The likelihood requires a probability model based on the observation data.

The pdfs $g_x(\xi \mid I)$ and $g_x(\xi \mid d, I)$ are called the 'prior' and 'posterior' pdfs, respectively. This terminology is used because Bayes' theorem is most commonly applied in a temporal context: prior to evaluating the initial state of knowledge, gather the data, and obtain the posterior state of knowledge under the probability model incorporated into the likelihood. In words, the full result is that:

$$\text{posterior distribution} = \text{likelihood x prior distribution}/\sum(\text{likelihood x prior})$$

where the denominator (the likelihood accumulated over all possible prior values) is a fixed normalising factor, which ensures that the posterior probabilities add up to 1;

$$\text{posterior distribution} \propto (\text{likelihood})\text{x (prior distribution)}$$

The relative influence of the prior data on the updated beliefs depends on how much weight we give to the prior (how informative make it) and the strength of the data. For example, a large data sample would tend to have a predominant influence on our updated belief unless the prior was extremely specific. If the sample ξ were small and combined with an informative prior, the prior distribution would have a relatively greater influence on the updated belief; this might be the case if a small observational study were combined with a prior based on a meta-analysis of previous findings. The flowchart of the Bayesian inference for measurement uncertainty quantification is depicted in Fig. 3.8. Univariate measurement model expressed as a relationship between output quantity Y and input quantities X on which Y depends, is defined by h in Fig. 3.8.

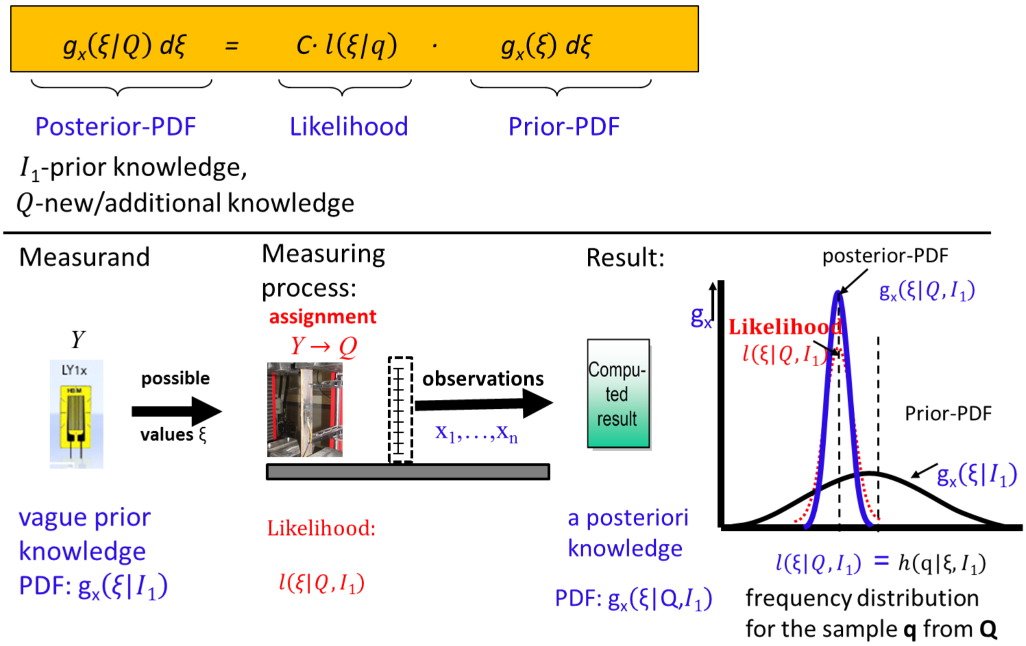


Figure 3.8: Flowchart of Bayesian inference for measurement uncertainty quantification

In this thesis, metrological results are assumed to be sampled from a normal probability model. From a Bayesian point of view, X and V are considered as two measurands, only the first of which is identified with a physical quantity. The relationship between X and V does not arise from the measurement model but rather from the probability model. Assuming that the probability model under which independent number of measurement n times, the results are the values $x = (x_1, ..., x_n)$ about quantity X are obtained is normal. Conditional on giving values x of X and ν of V, the pdf for any given datum x_i follows from the probability model and is:

$$L(x_i \mid x, \nu) \propto \frac{1}{\nu^{1/2}} exp\left[-\frac{1}{2}\frac{(x_i - \xi)^2}{2\nu}\right]. \tag{3.20}$$

X and V are also independent, and they act as spatial and scalar parameters, respectively, their joint prior separates into the product of a constant for the former multiplied by ν^{-1}

(Jeffreys' prior) for the latter. The joint posterior becomes:

$$g_x(\xi, \nu \mid x) \propto \frac{1}{\nu^{n/2+1}} exp\left[-\frac{1}{2\nu}\sum_{i=1}^{n}(x_i-\xi)^2\right]. \tag{3.21}$$

This expression can be conveniently arranged using the following identity

$$\sum_{i=1}^{n}(x_i-\xi)^2 = (n-1)s^2 + n(x-\bar{x})^2. \tag{3.22}$$

The mean $\bar{x}$ and standard deviation s^2 are calculated according to Eq. 3.4 and Eq. 3.5. Although the best estimate and associated uncertainty corresponding to the quantity V will rarely be needed, they can be derived using Bayesian interface. Then,

$$g_x(x, \nu \mid x) \propto \frac{1}{\nu^{n/2+1}} exp\left[-\frac{1}{2}\frac{(n-1)s^2 + n(x-\bar{x})^2}{\nu}\right]. \tag{3.23}$$

The expectation of this density is equal to zero and, which is Student's t density with $n-1$ degree of freedom, and its variance is

$$r^2 = \frac{n-1}{n-3}. \tag{3.24}$$

From the expressions for the expectation and variance given in the reference for density function, it can be concluded that the best estimate of the variance is:

$$\nu = (rs)^2 \tag{3.25}$$

with a standard uncertainty equal to the square root of:

$$\mu_\xi^2 = \frac{(rs)^2}{n}. \tag{3.26}$$

In this analysis, n does not need to be 'large'; two observations are sufficient. However, in order to obtain the standard uncertainty, n must be greater than three.

3.2.3 Monte Carlo Method

The MCM is a general tool for propagating distributions [7] through an input-output model $Y = f(X)$. The MCM carries the propagation of probability distributions sampling the probability distribution of the output quantity Y. In this method, PDF are explicitly assigned to all input quantities X_i based on information concerning these quantities. In some common circumstances, the JCGM [7] provides guidance on assigning PDFs to the input quantities X_i. In order to construct a PDF for a quantity based on a series of indications, Bayes' theorem can be applied. If appropriate information concerning systematic effects is available, the principle of maximum entropy can be used to assign a suitable PDF. Indeed, the JCGM [7] does not classify between Type A and Type B state-of-knowledge are probability distributions for the input variables. Nevertheless, this classification is still important [81] because the information that metrologists have for Type A and B quantities can differ greatly (statistical analysis or other means respectively).

A fundamental parameter for obtaining reliable results through MCM is the number M of trials or evaluations to be performed by the model. If M is chosen beforehand, there will not be direct control over the results. A value of $M = 10^6$ is often considered appropriate for providing a coverage interval of 95%; however, the random nature of the process and the nature of the probability distribution of the output quantity Y influence the value needed for M, which will vary in each case. The basic idea of this method is to draw random derivatives from the distribution $gx_1, ..., x_N(\xi_1, ..., \xi_N)$ and to propagate these deviations through the model shown in Fig. 3.2 to yield random deviates distributed of $gy(\eta)$. By repeating this many times yields an empirical distribution of $gy(\eta)$, which is used to determine the estimate y and the associated standard uncertainty $u(y)$ as well as coverage intervals. The calculation steps are shown in Fig. 3.6. The propagation of the probability density functions $g_i(\xi_i)$, $i = 1, ..., N$, for the values of the input quantities through the model to provide the probability density function $g(\eta)$ for the output quantity value is illustrated in Fig. 3.9 for the case $N = 3$. The model input quantities are $X = (X_1, X_2, X_3)^T$. The probability density functions $g_i(\xi_i)$, for X_i, $i = 1, 2, 3$, are rectangular, triangular, and Gaussian, respectively. The probability density function $g(\eta)$ for the value of the output quantity Y is asymmetric, which can be the case for non-linear models. An asymmetric $g(\eta)$ can also arise if the probability density functions for the values of the input quantities are asymmetric. This Figure is the counterpart of Fig. 3.5 for the law of propagation of uncertainty. Like the GUM, the JCGM [7, 31, 82] is concerned with models with a single output quantity.

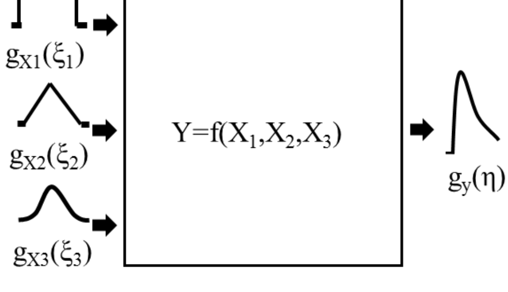


Figure 3.9: Illustration of the propagation of distributions for $N = 3$ input quantities

3.2.3.1 Computational Aspect for Univariate Models

The MCM method defines only joint probability density functions considered in the JCGM [7], which are univariate Gaussian. A univariate Gaussian probability density function with expectation $x = (x_1, ..., x_N)^T$ and uncertainty matrix V is given by

$$g(\xi) = \frac{1}{((2\pi)^N detV)^{1/2}} exp\left\{-\frac{1}{2}(\xi - x)^T V^{-1}(\xi - x)\right\}. \quad (3.27)$$

For the following equation, this probability density function reduces to the product of N univariate Gaussian probability density functions when there are no covariance effects. In that case,

$$V = diag(u^2(x_1), ..., u^2(x_N)), \quad (3.28)$$

where

$$g(\xi) = \frac{1}{(2\pi)^{N/2} u(x_1)...u(x_N)} exp\left\{-\frac{1}{2}\sum_{i=1}^{N}\frac{(\xi_i - x_i)^2}{u^2(x_i)}\right\} \quad (3.29)$$

$$= \prod_{i=1}^{N} g_i(\xi_i) \quad (3.30)$$

with

$$g_i(\xi_i) = \frac{1}{\sqrt{2\pi}u(x_i)} exp\left\{-\frac{(\xi_i - x_i)^2}{2u^2(x_i)}\right\}. \quad (3.31)$$

Monte Carlo simulation provides a general approach to numerically approximating the distribution function $G(\eta)$ for the value of the output quantity $Y = f(X)$ The relevant model and uncertainty propagation are defined in section 3.2.1. The model considers Eq. 3.2 and Eq. 3.3. Let the PDF for the ith input quantity $X - i$ be $g_{x_i}(\xi_i)$ and the PDF for Y be $g_Y(\eta)$. Let

$$G_Y(\eta) = \int_{-\infty}^{\eta} g_Y(z)dz \quad (3.32)$$

denote the distribution function (DF) corresponding to g_Y. An adequate approximation to $G_Y(\eta)$ will permit the determination of all required statistics associated with Y. The implementation of an MCM is given in the case of the model above. For instance, it will permit the evaluation of the standard uncertainty associated with an estimate y of the output quantity Y. The procedure is as follows:

- Select the number M of MCM trials to be made;
- Generate M vectors x_r by sampling from the PDFs for the (set of N) input quantities X;
- For each vector x_r, evaluate the model to give the corresponding value $y_r = f(x_r)$ of the output quantity;

- Calculate the estimate y of the output quantity and the associated standard uncertainty $u(y)$ as the mean and standard deviation of the model values $y_r, r = 1, ..., M$;
- Sort the values $y_r, r = 1, ..., M$, into non-decreasing, and use the sorted values to provide a discrete representation G of the distribution function $G_Y(\eta)$ for the output quantity;
- Use the discrete representation of the distribution function to calculate a 95% coverage interval for the output quantity.

3.2.3.2 Estimate of the Output Quantity and the Associated Standard Uncertainty

The mean $\overline{y}$ of the values $y_r, r = 1, ..., M$, of the output quantity is taken as the estimate y of the output quantity, and the standard deviation $u(\overline{y})$ of the values is taken as the standard uncertainty $u(y)$ associated with y. $\overline{y}$ is evaluated from following:

$$\overline{y} = \frac{1}{M}\sum_{r=1}^{M} y_r; \tag{3.33}$$

and the standard deviation $u(\overline{y})$ defined as following

$$u^2(\overline{y}) = \frac{1}{M-1}\sum_{r=1}^{M}(y_r - \overline{y})^2. \tag{3.34}$$

The updating procedure evaluates the estimate of the output quantity and the associated standard uncertainty, which avoids the need to store the complete set of model values [83].

3.3 Measurement Uncertainties & Model Assignment Uncertainty in Monitoring Models

When measuring the value of a measurand, a vague statistical concept of the measurand can result in inefficient analysis of uncertainty. The vagueness is caused by the fact that the value of the measurand is an unknown parameter such as population mean or median and that the measurement of this value is a random variable. The uncertainties of the input quantities determine the uncertainty of the measurand. The estimation of uncertainty model of the random variables is discussed in section 2.3. Type A measurement uncertainties are derived by assigning a statistical model to an observation, and Type B measurement uncertainties are derived by physically modelling the measurement process. Both types of uncertainties are evaluated using Bayes' theorem, the principles of probability calculus, and the rules for constructing prior probability distributions.

A mathematical model has been developed to simulate the influence of various parameters on strain measurement. The model takes into account the interaction between all input variables.

For such an application, the measurand is the mechanical strain, which is the sum of the amplifier strain E_{amp} and the apparent strain E_{app}. It is calculated using Eq. 3.35. Here, the mechanical strain is the measured strain in structures. The amplifier strain denotes the strain that is measured with the amplifier. The apparent strain is the strain that is caused by temperature effects in the strain gauge:

$$E_{mech} = E_{amp} + E_{app}. \tag{3.35}$$

3.3.1 Measurement Uncertainty Associated with Measurement Equation

The probabilistic treatment of the measuring errors requires a limit state function or response function. The response function Eq. 3.36 provides a solution for this. Based on the physical properties of the measurement process, the measurement equation is derived, and uncertainty models are introduced for the associated random variables. This derivation is based on the determination of Type B uncertainties [6]. Model uncertainty and assignment uncertainty are also introduced. Model uncertainty E_{mech}, which describes the uncertainty associated with physically formulating the problem, yields Eq. 3.36. The measurement uncertainty is based on a measurement equation developed by HBM [84] and [85]:

$$E_{mech} = \theta_{E_{mech}} + E_{amp} + E_{app}. \tag{3.36}$$

A measurement equation is a mathematical relationship between input quantities and an output quantity. This results in the measurement equation for the amplifier strain, in Eq. 3.37:

$$E_{amp} = f_{a,a} \frac{4}{k(1 + f_{s,v} + f_{s,s} + f_{s,q} + \alpha_{s,k}\Delta T_{20^\circ c})} \frac{U_A}{U_B} + f_{a,z}; \tag{3.37}$$

$$f_{s,q} = \frac{q(\varepsilon_q + \varepsilon_l \nu)}{\varepsilon_l(1 - q\nu)} \tag{3.38}$$

where k - factor, $f_{s,q}$ is the transverse strain correction factor, $\alpha_{s,k}$ is the temperature coefficient, $f_{s,v}$ is the gauge variation factor, $f_{a,a}$ is the amplifier deviation, $f_{s,s}$ is the model uncertainty of the gauge factor variation, and $f_{a,z}$ is the zero deviation. The transverse sensitivity error is a function of the corrected values of the strains on the parallel and perpendicular axes of the gauge, ϵ_a and ϵ_t respectively as well as the transverse sensitivity coefficient q and Poisson's ratio ν. If the ratio of the transverse to the longitudinal strain ϵ_q/ϵ_l differs from the calibration procedure, a correction according to Eq. 3.38 is considered depending on the transverse sensitivity of the strain gauge q. The resistance of the gauge can be changed by changing the temperature of the material in which the stain is being measured or by changing the temperature of the environment of the gauge. This apparent temperature-induced strain can become the main source of uncertainty of measuring system. The relationship between strain and temperature can be seen in Fig. 3.10. The tolerance of the temperature variation curve ϵ_T characterises the uncertainty of the apparent strain increasing with temperature:

$$E_{app} = \varepsilon'_{app}\Delta T_{20^\circ c} + \varepsilon_T \Delta T_{20^\circ C}. \tag{3.39}$$

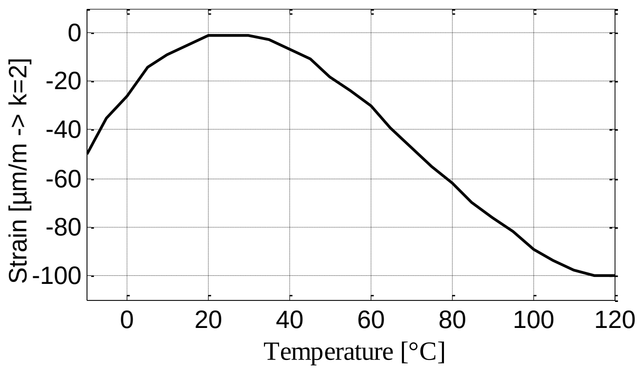


Figure 3.10: Relation between temperature and strain

Temperature-induced strain is determined by applying a much more general equation, which can be used to evaluate the thermal output if the conditions for using the graph and equation provided by the manufacturer cannot be met. The equation is as follows [86]:

$$\varepsilon'_{app} = \frac{\left[\beta_G + F_i\left(\frac{1+K_t}{1-\nu K_t}\right)(\alpha_S - \alpha_G)\right]\Delta T_{20^\circ C}}{k} \tag{3.40}$$

where β_G is the temperature coefficient of resistance of the grid conductor, α_S and α_G are the thermal expansion coefficients of the substrate and the electrical resistance of the conductive grid, respectively, $T_{\circ C}$ is the temperature at which the Wheatston bridge has been balanced to zero strain, and k is the gauge-factor setting of the measuring instrument. This equation takes into account the transverse sensitivity of the gauge in the term $(1 + K_t)/(1 - \nu K_t)$ because in isotropic materials, the thermal expansion/contraction will be uniform in all directions. The gauge factor refers only to the sensitivity of the strain gauge in a uni-axial stress state with a principal strain of $1/\nu$. The shape of this equation also cannot be assumed because the terms inside the brackets are functions of temperature [86]. This equation clearly shows that the thermal output depends not only on the characteristic of the strain gauge but also on the thermal and mechanical properties of the material to which the gauge is bonded.

In general, the mean μ_i and the standard deviation σ_i are associated with strain measurement i and the prescribed resistance threshold of the monitored physical quantity. The statistical descriptors can be obtained from measurement. The application of measurement Eq. 3.37 produces i measurement uncertainty models $M_{u,i}$ for i different strains given by Eq. 3.41:

$$E_{amp} \mid U_A/U_B \sim M_{u,i}(\mu_i, \sigma_i). \tag{3.41}$$

The parameters of this model μ_i and σ_i are considered as uncertain in order to account for the assignment uncertainty. A prior probability density function for the parameters is derived by calculating the individual probability density P_j by associating j different reference strains, which is given in Eq. 3.42. If the reference strain, $\epsilon_{R,j}$ of a normal distribution is the mean value, μ_i, the prior density $P_{\mu,i}$ can be modelled by a normal distribution with a mean μ_i' and standard deviation σ_i', i.e.

$$P'(\mu_i, \sigma_i \mid M_{u,i}) = P(M_{u,i} \mid \varepsilon_{R,j}); \tag{3.42}$$

$$\varepsilon_{R,j} = \frac{4}{k}\left(\frac{U_A}{U_B}\right). \tag{3.43}$$

The likelihood estimate of the parameter is derived from observations of the measurement and is defined by Eq. 3.44:

$$L(\mu_i, \sigma_i \mid \varepsilon_1 ... \varepsilon_n) = \prod_{j=1}^{n} P(\varepsilon_j \mid \mu_i, \sigma_i). \tag{3.44}$$

The posterior distribution of the parameters is derived via Bayesian updating, and marginal distribution of the mechanical strain is calculated with Eq. 3.45:

$$P_{Eamp}^{Poat} = \int_{-\infty}^{\infty} P(E_{amp} \mid \mu, \sigma) P''(\mu) P''(\sigma) d\mu d\sigma. \tag{3.45}$$

The uncertainty of the assignment is calculated using these steps and is contained in E_{amp}. The measurement Eq. 3.37 and Eq. 3.39 are inserted into Eq. 3.36, and the measurand can be determined.

3.3.2 Measurement Uncertainty Associated with Observation Equation

An alternative approach starts from the observation equation, which relates to the experimental data to the measurand. This allows a uniform treatment of the most diverse metrological problems. Once it is used in the context of Bayesian interface, it also facilitates the exploitation of any information provided by experimental data. Here, the amplifier strain and the apparent strain are determined based on observation ϵ using the definition of Type A measurement uncertainty [6, 7]. The measurement Eq. 3.35 is rewritten for distinguishing the different concepts of uncertainty determination and to account for the model uncertainty $\theta_{\epsilon mech}$ in Eq. 3.46:

$$\varepsilon_{mech} = \theta_{\epsilon mech} + \varepsilon_{amp} + \varepsilon_{app}. \tag{3.46}$$

It is assumed that n realizations of the amplifier strain ε follow a normal distribution with the parameters μ and σ in Eq. 3.47:

$$\varepsilon_{amp} \sim N(\mu, \sigma). \tag{3.47}$$

The parameters of the distribution are estimated using the method of maximum likelihood. The statistical uncertainties of the parameters are given by Eq. 3.48:

$$P_{\varepsilon_{amp}}(\varepsilon_{amp}) = \int_{-\infty}^{\infty} P(\varepsilon_{amp} \mid \mu, \sigma) P(\mu) P(\sigma) d\mu d\sigma. \tag{3.48}$$

Together with temperature data, the distribution of ε_{app} can be determined, which leads to the distribution of ε_{mech}, the measurement uncertainty. The measurement uncertainty obtained from observation equation has different boundary conditions associated with the probabilistic models. In contrast to the measurement equation based measurement uncertainty, the observation equation based measurement uncertainty applies to the type of sensor and amplifier used as well as to the specific application and surrounding conditions. In these circumstances, it is advantageous to express the measurement model as an observation equation (or as a system of simultaneous observation equations) instead of as a measurement equation. This means that focus is placed on the measurement data and that these are expressed as functions of the strain. It also implies postponing this to a subsequent phase of the analysis, whereby focus will be on the measured strain.

3.3.3 Measurement Uncertainty Associated with Bayesian Updating Approach

Uncertainty in an estimate of the value of a measurand is assessed by propagating the uncertainty in estimates of values of input quantities based on an equation that expresses the former values as a known function of the latter values. However, in measurement situations in which some of the input quantities depend on the measurand, this approach is circuitous and ultimately impracticable.

The observation equation applied to determine the measurement uncertainty must not necessary cover all surrounding conditions but at least those that apply for situations in which the observations are generated. This clearly constitutes a limitation of the observation-based measurement uncertainty, which is typically experienced during measurements. Measurement uncertainty based on the measurement equation should give an upper boundary; the measurement uncertainty based on the observation equation may give a lower boundary.

The Bayesian approach is particularly advantageous in situations in which both Type A and Type B uncertainty must be taken into account [6, 7]. It is also useful for harmonising information about a measurand derived from empirical data with information derived from other sources (e.g. expert knowledge or past experience).

Here, all participating quantities are modelled as random variables if there is uncertainty about their values – either because knowledge about them is imperfect or incomplete or because they are subject to the unpredictable vagaries of experimentation.

Measurement uncertainty based on the measurement equation can be viewed as the accumulation of the prior knowledge of the measurement process and therefore constitutes the prior measurement uncertainty given by Eq. 3.49:

$$P'(E_{mech}) = P(\theta, E_{amp}, E_{app}). \tag{3.49}$$

The posterior measurement uncertainty is based on the Bayesian updating. It utilises all available information and data, informative distributions for the prior, and the likelihood i.e. measurement uncertainty based on observation equation. The posterior measurement uncertainty based on the distribution of measurement uncertainty accounting for the prior knowledge and likelihood function is given by Eq. 3.50:

$$P''(E_{mech} \mid \varepsilon_{mech}) = \frac{P(E_{mech} \mid \varepsilon_{mech}) P'(E_{mech} \mid \varepsilon_{mech})}{\int_{-\infty}^{\infty} P(E_{mech} \mid \varepsilon_{mech}) P'(E_{mech} \mid \varepsilon_{mech}) d\mu d\sigma}. \tag{3.50}$$

This posterior measurement uncertainty is interpreted as an estimate of the measurement uncertainty for an application. It should be situated between the boundaries established by the measurement-equation-based and observation-equation-based measurement uncertainty. This is clearly based on the boundary condition of the model involved.

3.4 Sensitivity Analysis (SA)

Sensitivity analysis is an important part of metrology, particularly for the evaluation of measurement uncertainties. It affords the metrologist a better knowledge of the measurement procedure and facilitates the improvement of it. A tool for sensitivity analysis is developed in the GUM [7]. Terms such as influence, importance, ranking by importance, and dominance are all related to sensitivity analysis. On the other hand, the objective of uncertainty analysis is to assess the effects of parameter uncertainties on the uncertainties in calculated results.

Uncertainty analysis takes into consideration the behaviour of a measurement scenario with respect to the evaluation of measurement uncertainty, whereby parameters of probability distributions of input quantities are directly varied in order to examine the effect on the measurement uncertainty of the measurand. The necessary concept provided in the modelling is already described in section 3.2.1.2.

This section focusses on sensitivity analysis. It examines the influence of an individual quantity or part of the model on the combined measurement uncertainty (also in comparison with the influence of the other quantities). The most direct approach for sensitivity analysis is to fix specific influence quantities to a representative value (e.g. mean or median) in order to prevent its variation for simulation. This is given by the partial derivatives of the measurement mode for which the best value is computed. Recent implementations of MCMs, which where were encouraged by the JCGM [7], do not entail computing partial derivatives. Another sensitivity measure should be used, if possible, a global method that is more accurate than other ones [40, 41, 42, 43].

This section discusses selected applications of sensitivity analysis as well as the differences between them various. Saltelli et al. [40] provide a good overview of sensitivity analysis methods and define the distinction between:

- Factor screen to extract influential factors from a system with a large number of input quantities;
- Local sensitivity analysis to examine the local (point) influence of the influence quantities, usually using the partial derivatives approach;
- Analysis of global sensitivity to apportion the uncertainty to the influence quantities.

The sensitivity analysis and uncertainty methods have been compared by Allard and Fischer [87] and Mario et al. [88]. The methods are inherent and a feeling for the behaviour of the measurement scenario under investigation. Thus, the when choosing a sensitivity method many aspects of the problem (e.g. linearity/monotony of the measurement model, computational time required for one run, and the total number of runs required) should be considered.

3.4.1 Local SA: Sensitivity Coefficients of GUM

The advantage of local SA is that the computational effort required is much less than that required for factor screening of global SA because the method for local SA uses analytical methods and partial derivatives. According to the GUM approach, the SA indices are based on the partial derivatives that have been calculated when applying the LPU. It linearises the measurement model and reduces input quantities to their first two moments in order to evaluate the measurement uncertainty. The measurement model described in Eq. 3.2 and sensitivity coefficients can be calculated using the following Eq. 3.51.

$$S_i^{GUM} = \frac{(\frac{\partial f}{\partial x_i})^2.u^2(x_i)}{u^2(y)} \tag{3.51}$$

where the partial derivatives $\partial f/\partial x_i$ are taken as sensitivity coefficients in the GUM [6] as already seen in Section 3.2.1.2. The coefficients indicate how the output quantity estimate y varies with changes in the value of the input estimates $x_1, ..., x_N$. Kessel at al. [89] and Motra et al. [90] have introduced a coefficient of contribution for both uncorrelated and correlated input variables. It can be defined as:

$$w_i = \frac{(\frac{\partial f}{\partial x_i})^2.u^2(x_i)}{u^2(y)} = \frac{(\frac{\partial f}{\partial x_i})^2.u^2(x_i)}{\sum\limits_{j=1}^{N} \frac{\partial f}{x_j}.u^2(x_j)}. \tag{3.52}$$

It is assumed that the input quantities are mutually independent. This is the case in the example presented here. It describes the relation of a quantity X_i to the measurand Y.

3.4.2 The one At a Time Method (GUM-S1)

The JCGM [7] suggests this method for estimating sensitivity indices for an MCM. The computation of an associated sensitivity measure associated is not straightforward. For simulation, the sensitivity indices are evaluated by varying one quantity and keeping all others fixed. This method provides a quantification of effect to the given input quantity X_i on the standard deviation of the output quantity, which operates in terms of the actual non-linear model rather than a linearised counterpart.

This method implies making as many Monte Carlo simulations as the total of input quantities n, which may be quite heavy if n is large. It also allows for the evaluation of the sensitivity while varying only one input quantity at a time. However, in reality, all input quantities are varied at once. If interactions should arise in a measurement process, this will only be to the extent that all quantities involved were varied.

For comparison with the absolute value of linear sensitivity coefficients $c_i = \partial f/\partial x_i$ for a quantity X_i the non-linear coefficient can be used. The term $u_i(y)$ is the resulting uncertainty of a MCM, which is explained in section 3.2.3.

$$c_i = \frac{u_i(y)}{u(x_i)} \tag{3.53}$$

where only the quantity X_i is varied while the other quantities X_j, $j = 1, ..., N$ and $j \neq i$, are set to their best estimate. The standard uncertainty $u(x_i)$ of the quantity X_i can be obtained also using the MCM.

3.4.3 Variance-Based Sensitivity Indices

3.4.3.1 Definition

Global sensitivity refers to the global variability of an output over the entire range of the input variables of interest. It thus provides an overall view of the influence of inputs on an output as opposed to a local view of partial derivatives. This thesis mainly focusses on measurement uncertainty, which is estimated using the GUM, the Bayesian Method, and the MCM for evaluating the measurement model. Therefore, the examples presented here concentrate on variance-based methods i.e. methods focussing on the squared standard deviation.

The total variance decomposition theorem states that the variance of a quantity Y may be decomposed as the sum of the variance of the conditional expectation of Y given X_i and the expectation of the conditional variance of Y given X_i:

$$V(Y) = V[E(Y \parallel X_i)] + E[V(Y \parallel X_i)]. \tag{3.54}$$

The first term $V_i = V[E(Y \parallel X_i)]$ denotes the part of the variance of Y that is cause by the variations of the input quantity X_i, while the second term $E[V(Y \parallel X_i)]$ denotes the part of the variance of Y that is attributed to all input quantities except for X_i.

The first order sensitivity index is then defined as:

$$S_i = \frac{V_i}{V(Y)}. \tag{3.55}$$

A second-order sensitivity index can also be deduced from the variance because of the coupling of quantities X_i and X_j:

$$S_{i,j} = \frac{V[E(Y \parallel X_i, X_j)] - V_i - V_j}{V(Y)}. \tag{3.56}$$

Higher order sensitivity indices can be obtained in the same manner until n^{th} order. Of course, it would be difficult to compute them all. Moreover, the higher the order, the more likely the sensitivity index is to be null. Hence, it is possible to define a total order sensitivity index T_i, which includes all the effects of all orders in which the input X_i is involved.

$$T_i = S_i + \sum_{j \neq 1} S_{i,j} + ... + S_{1,2,...,n} \tag{3.57}$$

If an input quantity X_i does not interact with another input in a measurement process, then $T_i = S_i$.

In order to improve a measurement process, the total sensitivity index should be considered in the general case. If a contribution standard uncertainty is reduced, the influence on the standard uncertainty of the output will result from the corresponding first order sensitivity index. However, it will also result from the higher order sensitivity indices if the same input quantity is involved. This is the case for the examples presented here.

There are different methods available for estimating these quantities. The Sobol' method is presented here. A variation of this approach, which allows the total effect of a quantity on the measurement uncertainty to be investigated using less computational power, will also be introduced.

3.4.3.2 The Sobol' Sensitivity Index

The Sobol' method [41, 87] is used to quantify the sensitivity index associated with the evaluation of measurement uncertainty. This method allows the computation of first and higher order sensitivity indices as well as total order indicies [41]. To describe the Sobol' sensitivity estimate for two M-samples $(x_{i,j})_{i=1,\dots,n;j=1,\dots,M}$ and $(x'_{i,j})_{i=1,\dots,n;j=1,\dots,M}$, they are mixed together. The first order sensitivity index is then estimated by the quantity:

$$S_i^{Sbol} = \frac{\hat{D}_i}{\hat{D}} \tag{3.58}$$

where

$$\hat{D}_i = \frac{1}{M}\sum_{k=1}^{M} f(x_{1,k,\dots,x_{n},k}) f\left(x'_{1,k,\dots,}x'_{(i-1),k,}x_{i,k,}x'_{(i+1),k,}x'_{n,k}\right) - \hat{f}_0^2 \tag{3.59}$$

is the estimator of the variance of the conditional expectation of Y given X_i and

$$\hat{D} = \frac{1}{M}\sum_{k=1}^{M} f^2(x_{1,k,\dots,x_{n},k}) - \hat{f}_0^2 \tag{3.60}$$

is the estimator of the total variance of Y. The quantity

$$\hat{f}_0 = \frac{1}{M}\sum_{k=1}^{M} f(x_{1,k,\dots,x_{n},k}) \tag{3.61}$$

denotes the empirical mean of y.

$\hat{D}_{ij}$ may also be defined as above with both $x_{i,k}$ and $x_{j,k}$ taken from the first sample, while the other inputs are taken from the two samples and enable the estimation of the second order indices:

$$S_{i,j}^{Sobol'} = \frac{\hat{D}_{ij} - \hat{D}_i - \hat{D}_j}{\hat{D}}. \tag{3.62}$$

Each Sobol' index $S_{i,j}^{Sobol}$ represents a sensitivity measure that describes the amount of each variance $\hat{D}$ caused by the randomness of the single random input variables and the mapping of this onto the output variables. In practice, all partial sensitivity indices involving the single input variable $x_i (i = 1, 2, ..., n)$ are summed up to the total sensitivity index $T_i^{Sobol'}$ in Eq. 3.63 in order to evaluate the total effect of x_i. The total Sobol' indices thus consider the interaction between the input variables. Each Sobol' index $T_i^{Sobol'}$ represents the sensitivity of the measurement model with respect to input variable x_i without assuming linearity or monotonicity in the model:

$$T_i^{Sobol'} = 1 - \frac{\hat{D}_{-i}}{\hat{D}} \tag{3.63}$$

where:

$$\hat{D}_{-i} = \frac{1}{M} \sum_{k=1}^{M} f(x_{1,k,\ldots,x_n,k}) f\left(x_{1,k,\ldots,}x_{(i-1),k,}x'_{i,k,}x'_{(i+1),k,}x_{n,k}\right) - \hat{f}_0^2. \tag{3.64}$$

Such an estimation is computationally expensive. Many runs are required to reach the convergence of the estimation. Building on the recommendation of Tang et al. [92], the Latin hypercube sampling (LHS) method [93] was used to implement the Sobol' method. Overall, computing the first-order, second-order, and total-order sensitivity indices requires $n(m + 2)$ model evaluations, whereby n is the number of LHS and m is the number of parameters being analysed.

3.5 Reliability Analysis

Reliability relates the magnitude of the measurement error in observed measurements to the inherent variability in the 'error-free', 'true', or underlying level of the quantity (terms are used synonymously) between subjects. If the reliability is high, measurement errors are small in comparison to the true differences between subjects. Subjects can thus be relatively well distinguished (i.e. in terms of the quantity being measured). If the measurement errors tend to be large compared with the true differences between the subjects, reliability will be low; differences between the measurements of two subjects could be simply attributable to error rather than a genuine difference in their true values. This section will discuss how the reliability theory can be applied to a metrological model. Reliability theory is applied in the destructive testing of structural steel, concrete, soil, and monitoring models [91].

The limit state function $G = R - S$ is the origin of the formulation engineering structures, whereby R is the resistance and S is the effects. The variables R and S are random and are described by the mean value (μ_R, μ_S) and the standard deviation (σ_R, σ_S). In fact, G is the so-called safety margin $M = R - S$, see Fig. 3.11. In practice, neither R nor S are constant parameters. Therefore, the description of these takes into account the scattering of the loads on a component as well as the scattering properties of the materials. There are three possible conditions:

- $R > S$ permissible condition;
- $R = S$, limit state condition;
- $R < S$ Failure condition.

In the simplest case, the limit state function $g(X_i)$ can be the function of just two random variables R and S, whereby the first is the generalised structural resistance and the second is the generalised action or affect. If the generalised action or action effect is bigger than generalised resistance, failure occurs. Therefore, the probability of failure of the $R < S$ state is determined by evaluating the following integral:

$$p_f = P(R < S) = \int_{-\infty}^{\infty} f_s(x) F_R(x) dx \tag{3.65}$$

whereby $F_R(x)$ is the probability that the R is smaller than a predetermined value x. In Fig. 3.11, this is the area under the PDF of R to the left of x. Function $f(x)$ is the value of the PDF effect to S at the point x. The probability that both $R < S$, and $s = x$ is the product of $f(x)$ and $F_R(x)$. Because x is limited between $-\infty$ and $+\infty$, it can be assumed, as shown in Eq. 3.65, that the integral is formed within these limits. The shaded area represents the failure domain D. This is a convolution integral. Only in simple cases is the solution a closed one.

In structural engineering, the application of reliability theory is presented in Fig. 3.11 and

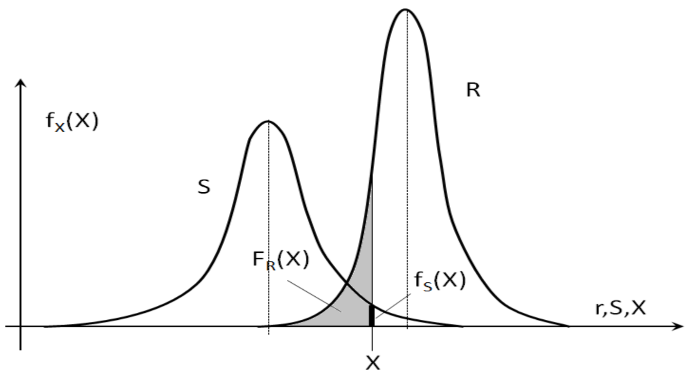


Figure 3.11: Descriptive representation of the Convolution integral according to Eq. 3.65 [94]

Fig. 3.12. The representation of the PDF for R and S and therefore the PDF generated for $R-S$ are shown. The failure region shown in Fig. 3.12 clearly corresponds to the p_f in Eq. 3.65 formulated by the integral.

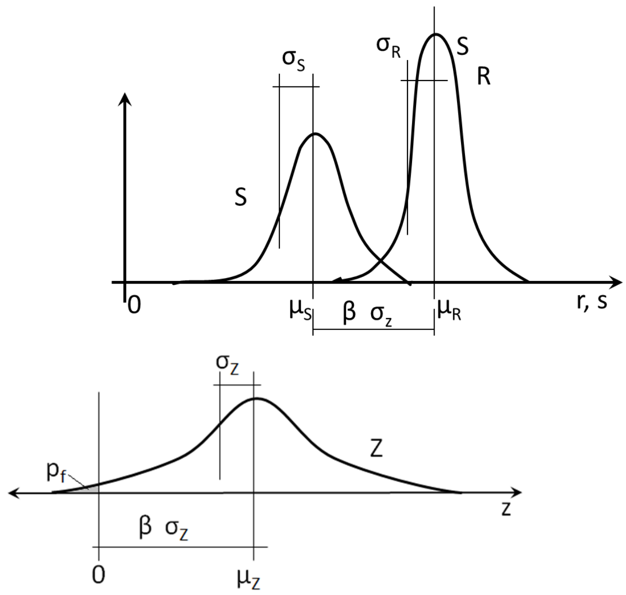


Figure 3.12: Top: PDF of resistance R and impact S. Bottom: PDF from transformation $Z = R - S$. The reliability index β can be read out as a multiple of σ_Z [94]

For normally distributed resistance and normally distributed effect, the mean value μ_Z and standard deviation σ_Z are defined as:

$$\mu_Z = \mu_R - \mu_S; \tag{3.66}$$

$$\sigma_Z = \sqrt{\sigma_R^2 + \sigma_S^2}. \tag{3.67}$$

The safety index β is represented as a multiple of the standard deviation σ_Z and lies to the right of the failure region p_f, as is shown in Fig. 3.12. Each value of β corresponds to a probability of failure, which in the case of a normal distribution for R and S via $p_f = \phi(-\beta)$ can be determined (for mean = 0 and standard deviation = 1).

The probability of failure p_f can be expressed by following equation:

$$p_f = P(R - S < 0) = P(Z < 0) = \phi\left(\frac{0 - \mu_Z}{\sigma_Z}\right) = \phi(-\beta). \tag{3.68}$$

For cases in which it is at the limit state function, $R - S$ is linear function, and R and S are normally distributed. As described above, p_f can be calculated by hand. If, however, the limit state function of several factors (basic variables) is not normally distributed, a simple calculation is not possible. First order reliability method (FORM) and second order reliability method (SORM) are used [95].

The weight of influence quantities in the measurement model is calculated as follows:

$$\alpha_R = \frac{\sigma_R}{\sqrt{\sigma_R^2 + \sigma_S^2}}; \alpha_S = \frac{\sigma_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}; \tag{3.69}$$

$$\alpha_R^2 + \alpha_S^2 = 1. \tag{3.70}$$

The problem can be illustrated graphically, as in Fig. 3.13 in which, marginal density functions $f_R(r)$ and $f_S(s)$ together with the joint density function $f_{R,S}(r,s)$ of two random variables are shown. The shaded area represents the failure domain D. For such a problem, the probability failure becomes:

$$p_f = P(R - S < 0) \int\int_D f_{R,S}(r,s) dr ds \tag{3.71}$$

when the basic variables R and S are independent (there is no any statistical correlation between them) Eq. 3.71 can be rewritten as follows:

$$p_f = P(R - S < 0) \int_{-\infty}^{\infty} F_R(x) f_S(x) dx \tag{3.72}$$

where $F_R(x)$ is the probability that $R \leq x$ and $f_S(x)$ represent the probability that load effect S takes the value between x and $x + \Delta x (\delta x \to 0)$. The integral over all possible x gives the total probability of failure.

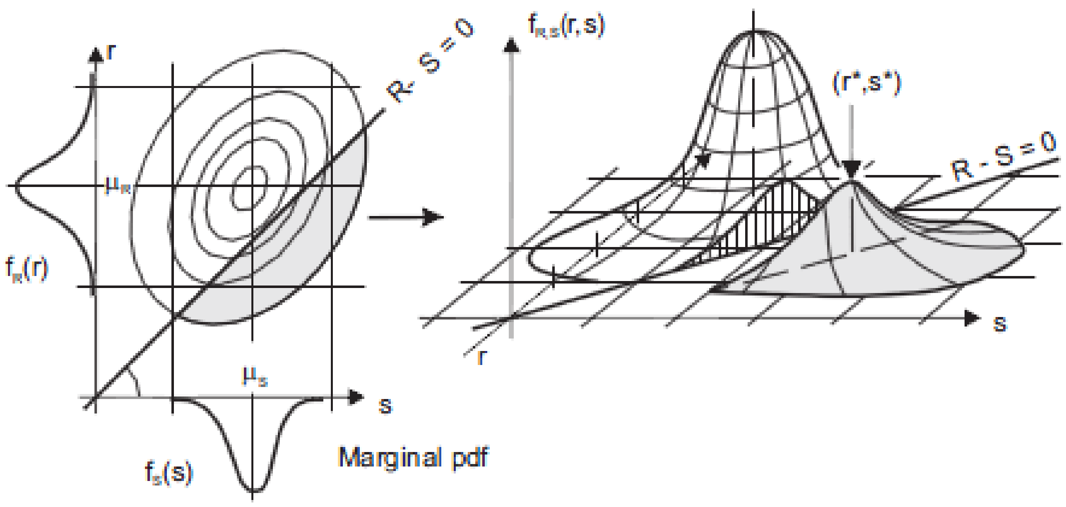


Figure 3.13: Two random variables joint density function $f_{R,S}(r,s)$, marginal density functions $f_R(r)$ and $f_S(s)$ and as shaded failure domain, from Schneider [94]

These measures of variability can be expressed as standard deviation (σ). Reliability is formally defined as:

$$\lambda = \frac{Var(X)}{Var(Y)} = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2} = \frac{\sigma_x^2}{\sigma_y^2} \tag{3.73}$$

where X_i: True value, Y_i: Measurement of X, U_i: measurement uncertainty.

If reliability is high, measurement uncertainties are small in comparison to the true difference between subjects. Subjects can thus be clearly differentiated based on the uncertainty-prone measurements. Conversely, if the measurement uncertainty is larger than the true difference between the subjects, the reliability will be low. The difference between the measurements of two subjects could simply arise from uncertainty rather than a genuine difference in the true values.

3.6 Robustness of Experiments

During the last decade, planned experiments have become a major tool for improving quality, which was largely stimulated by the quality engineering ideas of Taguchi [23]. A major goal of Taguchi's [23] experiments was to determine which design factors (i.e. controllable process parameters) lead to dispersion (i.e. affect variability) and figure out how to adjust them in such a way that minimises variability [96, 97]. Robust design refers to activities aimed at achieving that goal. One of the novel ideas in Taguchi's work on industrial experiments is the use of noise factors. These are impossible or too expensive to control during product manufacture or use. However, they can be set at fixed levels during an experiment and varied jointly with design factors. The use of noise factors can thus dramatically increase power for detecting factors with dispersion effects provided that the noise factors are explicitly modelled in the subsequent analysis [98]. In Fig. 3.14, it is shown that the robustness and optimal complexity of the model highly depend on the size and quality of the data.

The Taguchi method uses a loss function to determine the quality characteristics. Loss function values are also converted to a signal-to-noise (S/N) ratio (η). In general, there are three different quality characteristics in S/N ratio analysis i.e. 'the nominal is the better', 'the larger is the better', and 'the smaller is the better'. For each level of process parameters, the signal-to-noise ratio is calculated based on S/N analysis.

Nominal is the best;

$$\eta = S/N_T = -10log\left(\frac{\bar{y}^2}{s_y^2}\right) \tag{3.74}$$

Larger is better;

$$\eta = S/N_L = -10log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y^2}\right) \tag{3.75}$$

Smaller is better;

$$\eta = S/N_S = -10log\left(\frac{1}{n}\sum_{i=1}^{n}y_i^2\right) = -10log\left(\bar{y}^2 + \frac{n-1}{n}s^2\right) \tag{3.76}$$

where $\bar{y}$ is the mean of observed data; s_y^2 is the variance of y, n is the number of observations (degree of freedom) and y is observed data.

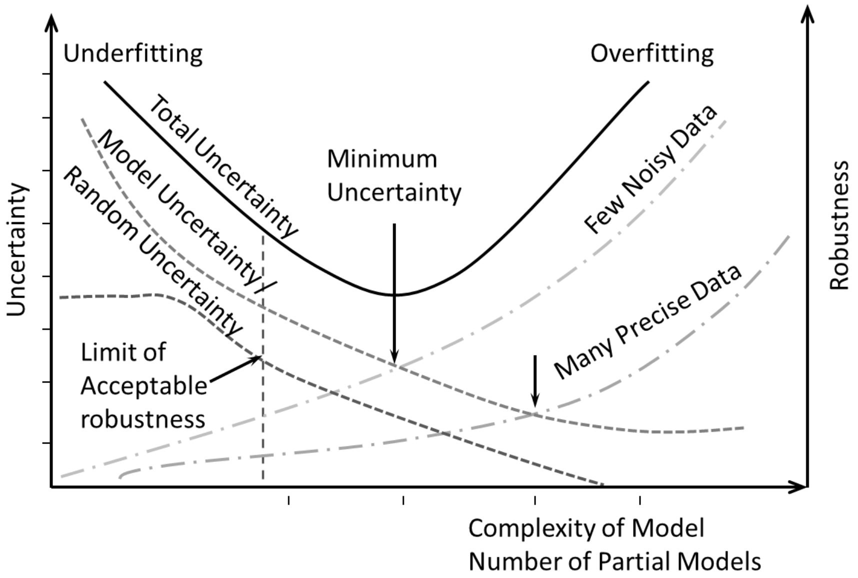


Figure 3.14: Scheme for predicting the uncertainty depending on the size and quality of the data set, which influence the estimation uncertainty

The equation of 'the nominal is the best' was selected for the calculation of S/N ratio because the nominal values of experimental inputs and roundness uncertainty were desirable in terms of good experimental design.

Robust design is one of the key design parameter on the performance of the system is less sensitive to small variation in these parameters. This is done based on the response of the experimental output, as shown in Fig. 3.15.

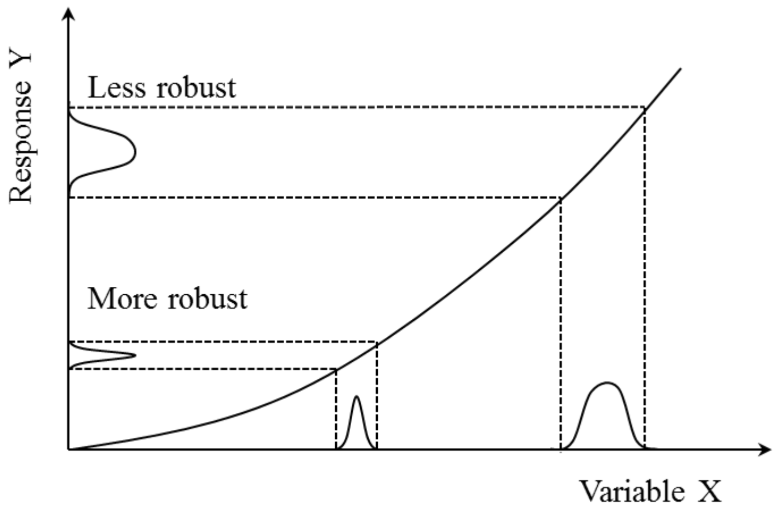


Figure 3.15: Exploring the nonlinearity of the experimental model to minimize the uncertainty of the response

3.7 Complexity and Cost Factor for Quality Assessment

The best measure for the generating ability is the error of prediction of as many independent separate validation data as possible. According to Fig. 3.14, the uncertainty of prediction is composed of two main contributions i.e. the remaining interference uncertainty and the estimation uncertainty. The interference uncertainty is the systematic uncertainty (bias) caused by unmodelled interference in the data; the calibration model is not complex enough to capture all the interferences of the relationship between sensor response and analysis. The estimation uncertainty is caused by modelling various kinds of random noise. The optimal prediction is obtained when the remaining interference uncertainty and the estimation uncertainty balance each other, as is shown in Fig. 3.14. If an overly complex model leads to increased prediction uncertainty, this is referred to as over-fitting. As seen in Fig. 3.14, the optimal complexity of the model highly depends on the size and quality of the calibration data set. For data sets that are noisy and limited in size, a simple calibration model is needed in order to prevent over-fitting.

The two curves in the Fig. 3.16 represent the costs inflicted by low quality data and the costs of maintaining high data quality. The costs inflicted by low quality data are, for example, faulty decisions based on low-quality data quality (which may or may not be based on experimental design). The costs of ensuring and maintaining high data quality simply refer to the work of assuring or improving data quality. The total costs associated with data quality are the aggregated cost of these two curves. From Fig. 3.16, it can be seen that assuring data quality would have the greatest effect i.e. the costs inflicted by low quality data decrease exponentially. Furthermore, the costs of ensuring high data quality are not casually related to their importance. Thus, the cost of assuring data quality directly increases with the cost data quality and assurance.

Interrelated components are generally quantified using the complexity and cost of different systems. Moreover, measuring the complexity of the experimental model complexity entails taking into account the number of parameters or simply PMs, the functional form, the range of the parameters, and the parameter prior. The most complex model has the maximum information regarding the response and is therefore the most beneficial for the user. The optimal complexity of the model highly depends on the size and quality of the data (Fig. 3.14 and Fig. 3.16). For data sets that are noisy and limited in size, a simple model is needed to prevent the increased prediction error resulting from an overly complex model. Both the cost of experiments and the quality of data increase with the increasing number of PMs. Standardised complexity is calculated as follows:

$$K_{m.norm} = \frac{n_{PM,m}}{n_{PM,max}} \tag{3.77}$$

where $n_{PM,m}$ represents the number of PMs of the considered experimental model m and $n_{PM,max}$ represents the number of PMs of the most complex experimental model.

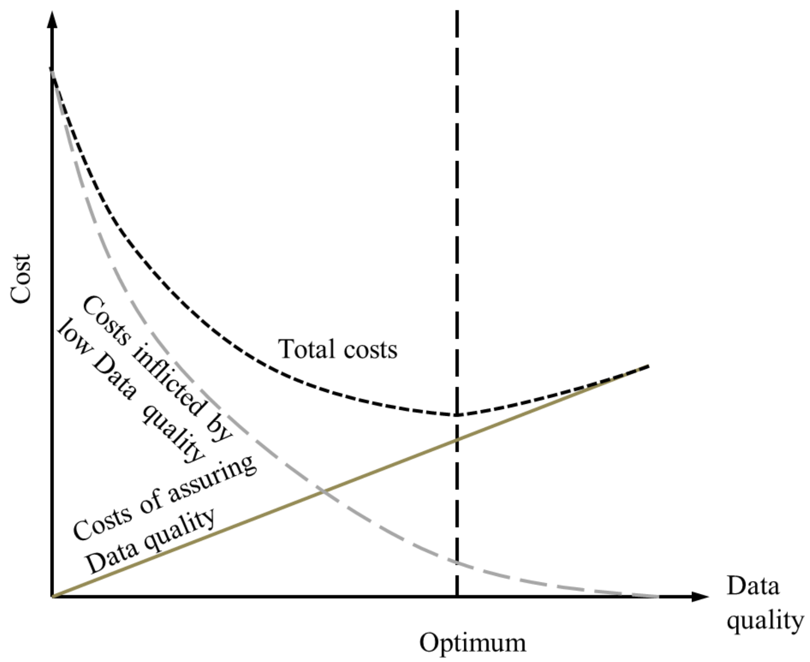


Figure 3.16: Total costs incurred by data quality on the experiment

3.8 Inter-laboratory Quality Assessment

Inter-laboratory studies (especially 'proficiency testing) are conducted to ensure measurement capability for commerce, to evaluate national and international equivalence of measure, and to validate measurement devices/methods or standard materials. In many inter-laboratory studies, a pilot lab prepares materials or objects to be measured and delivers them to participating labs. It is usual for one or more laboratories to report measurements that differ from the majority. There is no valid method for handling such differences in a statistical analysis. Methodologies for analysing inter-laboratory studies that model potential laboratory deviation must therefore be developed in such a way that prevents them from dominating the estimation of a measurand.

According to the DIN [99] accreditation, GRK 1462 feels constrained to continually increase its quality management system and professional competence. The determination of measurement uncertainty became an important topic in laboratory work. The models for estimating measurement uncertainty in testing are discussed in section 3.2. These up-to-date calculation models are based on actual testing, standard, updated pre-standards, related guidelines, or the literature.

The results of proficiency testing (including statistical distribution) can be represented in various ways. To evaluate the data, the appropriate statistical methods must be chosen because there is no standardised method for evaluating proficiency testing. To evaluate and assess the results of a proficiency testing, the following four steps are implemented [100]:

- Review and assessment of homogeneity, and stability of testing;
- Determination of mechanical properties and its uncertainty;
- Calculation of performance using statistics;
- Evaluation of performance.

3.8.1 Homogeneity Testing

Uniformity is a state in which a uniform structure composition has one or more predetermined characteristics. A reference material is deemed to be uniform in terms of a certain property if the given value lies within the specified uncertainty limit. The random homogeneity is evaluated using a statistical significance test (hypothesis testing). However, each statistical test method has some mathematical deficiency, which depends on the sample size. When applying test methods for the sample size examined, the null hypothesis of the material being homogenous is not refuted [101]. A more sensitive test method and/or a higher sample size may lead to a rejection of the null hypothesis. The homogeneity of a material can therefore never be proven. It can only be determined at a defined level of significance or non-homogeneity. The uncertainty of the materials value X, μ_x is calculated from Eq. 3.78:

$$\mu_x = \sqrt{(\mu_E)^2 + \left(\frac{S_{hom}}{\sqrt{n_{hom}}}\right)^2 + \left(\frac{ms}{2\sqrt{3}}\right)^2} \tag{3.78}$$

where, μ_E is the uncertainty of a single measurement, S_{hom} is the standard deviation of the homogeneity test, n_{hom} is the number of tested/used samples, and ms is the measuring step.

The mean and standard deviations were calculated for the results of all individual experiments. An F-test was used to determine equal or unequal variances between the groups, and a two-sample t-test was used to determine the significance in the results. A t-test with a p-value less than 0.05 (one-tail) was considered statistically significant, as asown in Fig. 3.17 and Fig. 3.18.

A negligible degree of heterogeneity may be preferable to a significant degree of uncertainty (Fig. 3.18, [101]) because there may be doubts about the effective degree of heterogeneity.

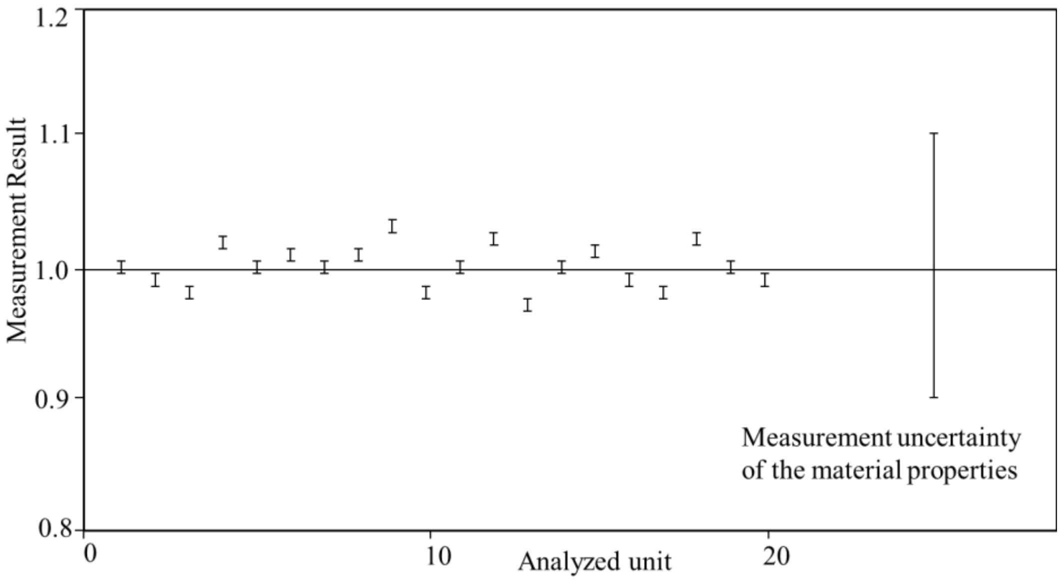


Figure 3.17: Example of a homogeneity study in which the detected heterogeneity is significant ($F_{exp} > F_{0.05}$) but negligible compared to the uncertainty interval of the certification measurement

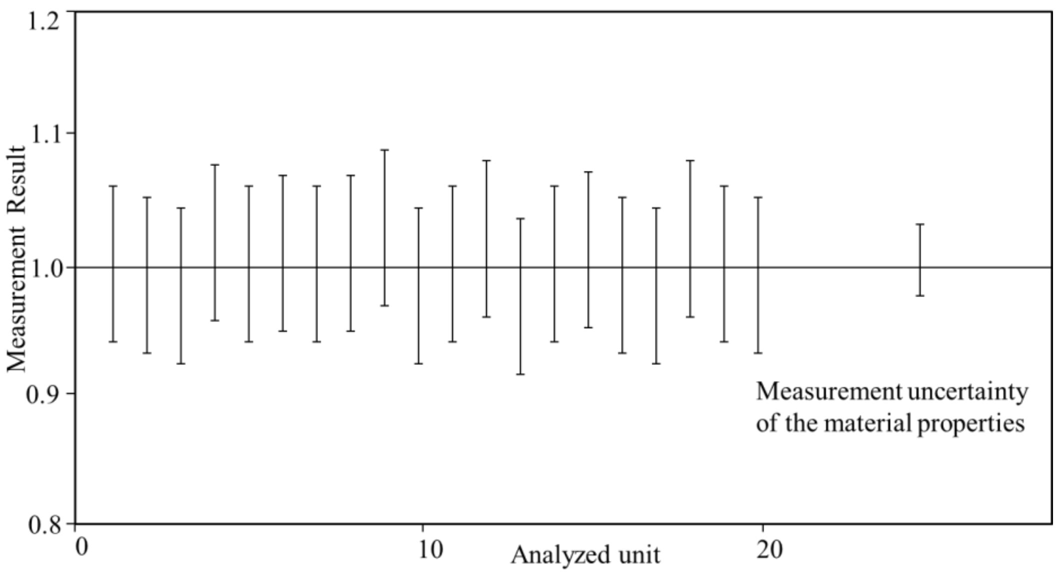


Figure 3.18: Example of a homogeneity study in which no heterogeneity can be detected ($F_{exp} < F_{0.05}$) but possible heterogeneity may be significant compared to the uncertainty interval of the certification measurement

3.8.2 Statistical Analysis

The statistical evaluation of the experimental results was based on the ISO 13528 [102] and the DIN EN ISO/IEC 17043 [103]. Because of an increased understanding of measurement uncertainty, it is seeing increased use in the evaluation. The measurement uncertainty of the assigned value and the measurement uncertainty of the participant result can be taken in account. If the standard uncertainty μ_x of the assigned value is much larger than the standard deviation for proficiency testing σ, t there is risk that some laboratories will receive action and warning signals because of inaccurately determined value and not laboratory error. The standard uncertainty of the assigned value shall therefore be established with maximum limit of 5 GPa in Eq. 3.80 [102] and must be met:

$$\mu_x \leq 0.3\sigma \leq 0.1 limit^* \tag{3.79}$$

where, σ standard deviation for proficiency testing.

The allowable deviation is defined as the limit*

$$limit^* = 3\sqrt{(\mu_x)^2 + \left(\frac{limit}{3}\right)^2}. \tag{3.80}$$

Depending on its interpretation, standard value (z-score) indicates the performance of a participant (i.e. satisfactory, questionable, or unsatisfactory). Z-score (the most commonly used score, measurement uncertainty is not taken into account):

$$z = \frac{x - X}{\sigma} \tag{3.81}$$

where, x is the result reported by participant; X is the assigned value; σ is the standard deviation for proficiency assessment.

z'-score (standard uncertainty of the assigned value is taken into account);

$$z' = \frac{(x - X)}{\sqrt{\sigma^2 + \mu_x^2}} \tag{3.82}$$

where, μ_x the standard uncertainty of the assigned value X.

zeta-score (standard uncertainty of the assignment value and the participants result is taken into account);

$$\zeta = \frac{(x - X)}{\sqrt{\mu_x^2 + \mu_X^2}} \tag{3.83}$$

where, μ_x is the participant's own estimate of the standard uncertainty x; μ_X is the standard uncertainty of the assigned value X.

En number (expanded uncertainty of the assigned value and the participants result is taken into account)

$$E_n = \frac{(x - X)}{\sqrt{U_x^2 + U_{ref}^2}} \tag{3.84}$$

where, U_x is the expanded uncertainty of the participant's result x; U_{ref} is the expanded uncertainty of the assigned value X determined in a reference laboratory.

The following judgment is commonly made for $z - z'$ and zeta score:

- $|z| \leq 2.0$ the score indicated "satisfactory" performance and generates no signal;
- $2.0 < |z| < 3.0$ the score indicates "questionable" performance and generates a warning signal;
- $|z| \geq 3.0$ the score indicates "unsatisfactory" performance and generates an action signal.

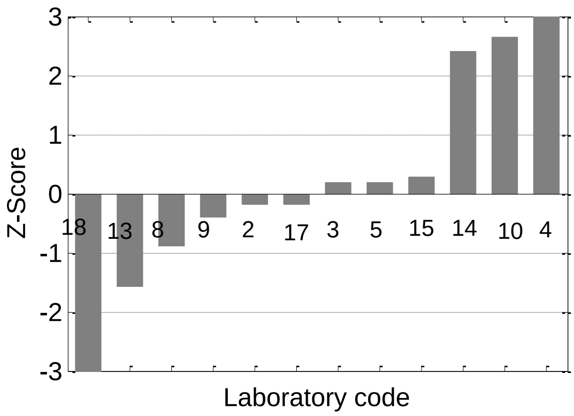


Figure 3.19: z-score bar chart of thickness of sample and tensile strength result of steel samples based on the Tensile Testing of Metals Proficiency Testing Program [104]

The following judgment is commonly made for E_n number:

- $|E_n| \leq 1.0$ the score indicates "satisfactory" performance and generates no signal;
- $|E_n| > 1.0$ the score indicates "unsatisfactory" performance and generates an action signal.

Fig. 3.19 shows the results for just those laboratories that give z-score within the range ± 3.0. The measurements of specimen thickness s for laboratories 1, 6, 7, 11, 12, and 16 that had a high degree of uncertainty have not been included in the figure.

Chapter 4

Novel Methodology for the Quality Assessment of Experimental Models

4.1 Introduction

The measurement infrastructure for materials is highly complex. There is therefore a lack of internationally recognised compatibility for measuring key properties based on a sound understanding of the behaviour of materials. The experimental model is a convenient way to increase the awareness of the need for a rigorous approach to measuring the properties of materials properties, for an improved understanding of the sources of variability, for the development of improved measurement methods, and for their harmonisation for international acceptance. This would improve the quality and reliability of materials data and generate a recognised framework for the measurement of materials [105].

This chapter is structured according to the typical modelling workflow of measurements. Establishing a complete comparative modelling pipeline was a basic prerequisite for dealing with loop prediction, different types of models, their relationship to the experimental model, and the assessment of quality assessment, which are described below. This chapter discusses the use of statistical analysis in assessing the output of the experimental models and experimental data. The assessment is performed in three parts; the first is based on a qualitative analysis of the output, which includes theory generation, problem identification, and the design of experiments. Qualitative assessment aims to improve the quality of experimental models. The second part is based on the quantitative analysis of experimental models. Different statistical methods are used to quantify the quality of models. The third part is based on the calculation of weighted for reliability and quantify the quality of output.

4.2 Evaluation Algorithm

Evaluation methods as a tool for objective assessment of experimental models should be developed in line with clearly defined objectives and requirements. The Experiemntal Model Quality Assessment (EMQA) is therefore based on the following characteristics of experimental requirements:

- Clear hypothesis;
- Dimensions and dimensional homogeneity;
- Similitude;
- Flexibility in terms of knowledge level and fabrication;
- Flexibility with the aspect of instrumentation;
- Repeatability and validity;
- Observation trends in the data, e.g. linearity;
- Applicability.

In the field of engineering, various models are available for abstract representation. The limitations and difficulties associated with constructing these models are apparent. The EMQA therefore includes a two-stage evaluation algorithm. After the objective experimentation is defined, a qualitative assessment takes place in the first stage in order to reduce the number of models. These are then quantitatively evaluated in the second stage of the EMQA.

4.3 Qualitative Assessment

Because of the need to rapidly produce reliable information, qualitative methodologies for quantifying uncertainty are seeing increased use in laboratories. Qualitative methods also have important advantages such as lower cost, speed, simplicity, or the minimisation of uncertainty caused by delays between sampling and analysis. In order to ensure that the method is fit for its purpose, typical performance characteristics such as accuracy, robustness, measurement uncertainty, complexity, and reliability must be known. Qualitative uncertainty is acknowledged, but most research on the quality of models has focussed on analysing quantitative uncertainty such as model parameter uncertainty, model uncertainty, and natural randomness [12, 13, 14, 15]. Few studies have discussed qualitative uncertainties [106, 107]. One example is Bayes' theorem, which was used to evaluate the quality of experimental models [105]. Among these studies, the discussion of qualitative uncertainty is limited. However, consideration of qualitative uncertainty is important for estimating the interior states of a system and promoting careful decision making [90]. To foster a complete understanding and reduce uncertainty, both quantitative and qualitative uncertainties need to be estimated concurrently. Unfortunately, in most cases, qualitative uncertainty has been neglected.

The methodology for qualitatively characterizing the uncertainty of the EMQA consists of two basic steps:

- Specification of uncertainty sources;
- Qualitative characterization of uncertainty.

The sources of uncertainty in experimental model are described in section 3.2. The nature and extent of the qualitative characterisation of these sources of uncertainty depend on the objectives of the EMQA and the appropriate form of output for its intended purpose. Prior to developing an assessment, its intended purpose must be clearly formulated.

The qualitative characterisation of uncertainty entails qualitatively evaluating the level of uncertainty of each specified source, defining the major sources of uncertainty, and qualitatively evaluating the appraisal of the knowledge base of each major source. The extent to which each item is considered depends on the nature of the relevant database and the purpose of the EMQA. The objective is to identify the sources of uncertainty that are most influential in determining the outcome of EMQA. Expert judgment is required to determine the qualitative relationship between sources.

If the magnitude of uncertainty is low and the values needed for the assessment are known, it was hypothesised that large changes within the source of uncertainty would only have a small effect on the results of the assessment. A designation of medium implies that a change within the source of uncertainty is likely to have a moderate effect on the results and that the values of the data sets needed for the assessment are unknown (completely or partially). A designation of high implies that a small change in the source would have a large effect on results and that the values of the data sets needed for the assessment are unknown.

The overall characterisation and evaluation of uncertainty of structural steel testing model are described in section 5.2 and the qualitative evaluation of tensile properties of steel is presented in Tab. 4.1. Where considerable variation of components prevented collection, a possible range (e.g. medium–high) was given. Before the sources of uncertainty can be assessed, low, medium, and high variation must be defined.

Based on this qualitative evaluation, it is clear that the factors influencing tensile steel

Table 4.1: Qualitative assessment of influence of specimen size, orientation, strain rate with rectangular cross-section on the tensile properties of hot rolled structural steels

	Influence parameter on tensile properties						
Mechanical properties	L_0	b_0	t_0	L_0/b_0	b_0/t_0	Orientation	Strain rate
Modulus	Low	Low	Low	Low	Low	High	High
Yield strength	Low	Low	Low	Low	Low	Medium	Medium
Tensile strength	Low	Low	Low	Low	Low	Medium	Medium
Elongation at maximum force	High	High	High	High	High	High	High

experimental model are much more complex than those described in the conceptual model. However, because of limited data, simplification was required. More detailed information on the underlying EMQA and a description for each of the judgments is presented in Tab. 4.1. The EMQA hypothesis based on EM complexity and data quality is presented in Fig. 4.1.

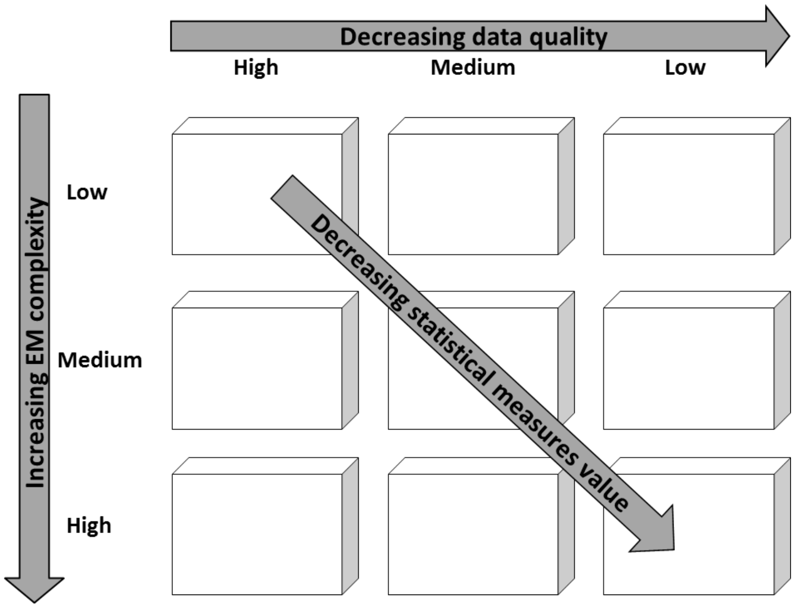


Figure 4.1: Direction of interaction data quality, EM complexity, and the statistical effect on EM

4.4 Quantitative Assessment

EMQA and data analysis can be of great value when attempting to draw meaningful results from a large body of qualitative data. One of the most intriguing and persistent problems in the evaluation of measurement models and data is how to combine the results obtained from several measurements of the same quantity. Here the problem is extracting a good estimate of the quality from the data. Most often, any kind of mean will do, and different means (e.g. unweighted and weighted) will generate similar values. The big question, however, is how to estimate the uncertainty of the respective mean. For EMQA, a reasonable estimate of uncertainty will need to account for the bias observed, and this task requires an appropriate mathematical model as a basis for EMQA. Depending on the experimental model and data evaluation techniques, uncertainty estimates may vary considerably. The terms agreement, reliability, reproducibility, and repeatability are also used with varying degrees of consistency. These may be useful parameters comparing two different methods. Thus, focus is on developing a methodology to quantitatively assess the quality of experimental results. The exemplary implementation is the aim of GRK 1462 Phase II. Furthermore, quantitatively assessing the behaviour and performance of materials is essential for the quality and reliability of products.

Once developed, measurement methods, would then be used on a range of materials in order to prove and improve the methods, thereby leading to a number of activities:

- Assessment of the quality of the measurements (accreditation and traceability);
- The validation of the method;
- Harmonization;
- Prediction.

The proposed quality factors are summarised in Fig. 4.2. The set of quality factors together incorporate the needs of all experiments and define a complete picture of the quality of the experimental model. The quality factors defined may be used as criteria for evaluating the quality of individual experimental models and comparing alternative representations. The definitions of the quality factors are:

- The evaluation of measurement uncertainty is necessary to report the quality of a measurement together with the value of a measurement;
- Sensitivity analysis is an important part of metrology, particularly for evaluating measurement uncertainties;
- Robustness has great potential in the area of low cost and high quality experimentation;
- Reliability relates the magnitude of the measurement error in observed measurements to the inherent variability in the 'error-free', 'true', or underlying level of the quantity (terms are used synonymously) between subjects.

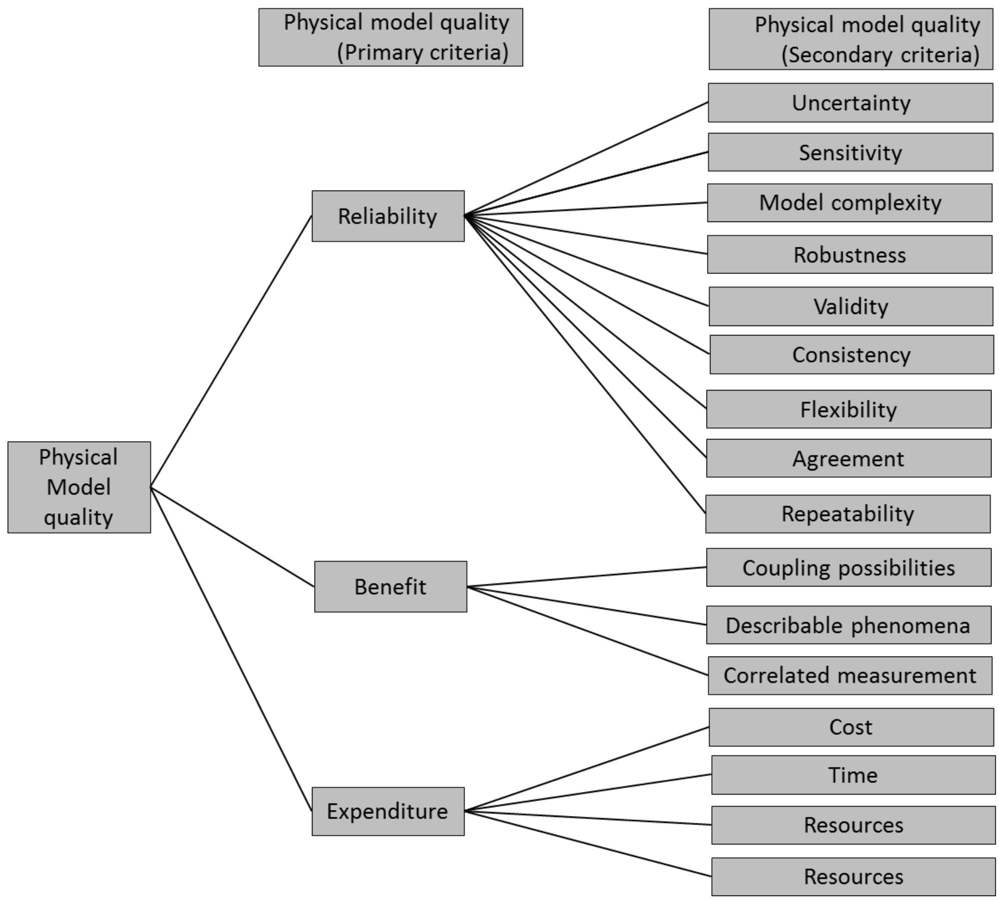


Figure 4.2: Model quality factors based on Keitel [26], Rueter [27], and Motra et al. [105]

4.4.1 Uncertainty Analysis

Probability distributions are the preferred way of expressing measurement uncertainty. Typically, such a distribution is the result of statistical inference based on a statistical model that expresses how experimental data relates to the measurand.

In section 2.1.2, it was stated that uncertainty can be regarded as a quantitative indication of the quality of a physical model with the aim of assessing the measurement uncertainty of entire experimental model (see section 3.2). Sections 3.2.1, 3.2.2, and 3.2.3 describe three approaches to calculating measurement uncertainty $\mu(y)$ based on the propagation of distribution functions. This is intuitively correct; if several experimental models of the same quality are available, the with a smaller uncertainty will be better than the others.

4.4.1.1 Weighted Calculation for Quality Assessment

The criteria of a technical system are not equally important. Depending on the specific requirements, the criteria may vary. The total experimental measurement uncertainty $U_E(y)$ is calculated according to Eq. 3.15, Eq. 3.26, Eq. 3.34, and Eq. 3.78. In this section, the EMQA of model j is defined depending on its uncertainty and the model with the lowest uncertainty at each parameter combination η:

$$EMQA(\eta) = \frac{min\,(U_E(\eta))}{U_E(\eta)}. \tag{4.1}$$

4.4.1.2 Experimental Models Quality Assessment

In the EMQA, there are many sources of measurement uncertainty; the influence of these should be considered. However, it is currently difficult to quantify the effect of the sources of uncertainty for all PMs. When quantifying the quality of the model, it is assumed that the EM with the lowest total uncertainty corresponds to the highest quality. Regarding the uncertainty and uncertainty range in Tab. 4.2, all models are evaluated equally according to Eq. 4.1 i.e. quality of Model 1 was higher than that of Models 2 and 3. This is because the uncertainty and scatter interval were lower than these models.

Table 4.2: E-Modulus EM quality for steel S355 [11]

Model	Mean value [GPa]	Uncertainty [%]	Scatter range [GPa]
Model 1	210.80	8.90	190.70–234.20
Model 2	208.50	12.50	190.30–249.40
Model 3	209.80	11.30	193.70–252.50

Several experimental models should be comparared to determine which is most likely to represent the best quality. The uncertainty for the models as well as confidence interval is obtained using the GUM, Bayesian, and MCM methods. The total measurement uncertainties of several experimental models (1, 2, 3, and 4) associated with estimated value of the tensile strength of steel are listed in Tab. 4.3. The data and related experimental models are presented in section 5.2. Based on the data presented in Tab. 5.2, it is clear that model 4 and the uncertainty estimated using MCM are smaller than the other methods. The methods for calculating measurement uncertainty are compared based on the uncertainty in the experimental model. Probability distribution is the preferred method for expressing measurement uncertainty. Such a distribution is typically the result of a procedure for statistical inference based on a statistical model that expresses how experimental data relates to the measurand.

4.4.2 Sensitivity Analysis

Sensitivity analysis is another way of analysing individual measurement scenarios in detail to determine the influence of a single quantity on the overall measurement uncertainty. For

Table 4.3: Comparison of the results of the expanded uncertainty for the tensile strength using GUM, Bayesian and MCM

	Estimates [MPa]			Total uncertainty [MPa]		
Model	GUM	Bayesian	MCM	GUM	Bayesian	MCM
Model 1	493.18	492.22	492.28	10.12	10.78	10.51
Model 2	487.87	487.31	487.09	11.04	11.90	11.10
Model 3	489.09	488.73	488.93	10.15	9.00	9.60
Model 4	490.20	491.33	492.07	10.10	10.00	9.68

sensitivity analysis, sensitivity coefficients are the key quantities that have to be evaluated. They are determined and assembled using the different methodologies described in section 3.4. When multiplied by the variation of the corresponding input parameter, they will quantify the influence on the targeted quantities. The results of the different approaches must now be analysed in practice, advanced real life measurement scenarios must be applied. It would also be interesting to implement and compare various other approaches.

In this thesis, some good practices in performing a SA associated to the evaluation of measurement uncertainty have been identified. In the application examples, the quality assessment is based on an uncertainty budget associated with the evaluation of measurement uncertainty.

Sample calculation of uncertainty sources (Type B) are shown in Tab. 4.4 for yield strength determination. Other sources of uncertainty (e.g. extensometer, gauge length of specimen, and repeatability), which have a sensitivity coefficient of 1.00 are not included in this table. From Tab. 4.4, it can be seen that the area of specimen is the main factor of total measurement uncertainty in yield of strength. The sensitivity indices of input parameter are nearly similar for all sensitivity approaches. Indeed, all three approaches can be used to determine measurement uncertainty.

Table 4.4: Result of sensitivity indices for Yield strength of S255 steel [105]

Source of uncertainty	Sensitivity coefficient		
	GUM [-]	OAT [-]	Sobol' [-]
Force	0.44	0.43	0.45
Mean area	0.66	0.67	0.65

4.4.3 Reliability Analysis

Reliability relates the magnitude of the measurement uncertainty in observed measurements to the inherent variability in the 'error-free', 'true', or underlying level of the quantity between subjects. These measures of variability can be expressed as standard deviation (σ), and reliability is formally defined as:

$$\lambda = \frac{Var(X)}{Var(Y)} = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2} = \frac{\sigma_x^2}{\sigma_y^2} \tag{4.2}$$

where X_i: True value, Y_i: Measurement of X, U_i: measurement uncertainty.

If reliability is high, measurement uncertainties are smaller than the true difference between subjects. The subjects can thus be clearly differentiated based on the uncertainty-prone measurements. Conversely, if measurement uncertainty is larger than the true difference between subjects, reliability will be low because difference between the measurements of two subjects could be attributed to the uncertainty rather than a genuine difference in the true values. The EMQA is defined by its reliability with the maximum reliability at each parameter combination:

$$EMQA(\eta) = \frac{max\,(\lambda(\eta))}{\lambda(\eta)}. \tag{4.3}$$

The measure of EMQA is a reference criterion for comparing the quality evaluation of different experimental models. The experimental model with the highest quality is recommended. Furthermore, the determined total uncertainty and reliability of the experimental output of each model can be considered in experimentation for estimating model parameters and validating a simulation model.

All theoretical background related to reliability is explained in section 3.5. This thesis not only considers the reliability of the output data but also the reliability of the measuring instruments. This thesis discusses how to express the reliability of the measurement referring to the concept of metrological traceability and the associated measurement uncertainty

4.4.4 Robustness Analysis

The analysis of uncertainties is related to the robust experimental design and the reliability of experimental model and data [108, 109, 110]. With a robust experimental design, is the system is less sensitive to small variations in these parameters. The Tahuchi [23] method is implemented for the tensile steel test to select the best input parameter. The detailed methodology is described in section 3.6. The optimal locations of sensor under parametric uncertainty should be studied in the reference object (i.e. concrete pole). In contrast, because of limited time and man power, the location of sensor in the concrete poles shown in section 6.3

is decided on the basis of experience and expert knowledge. Research on the optimal location of the sensor in monitoring models is being conducted on GRK 1462. At the end (2017) of GRK 1462, the methodology for the optimal locations of sensor under uncertainty will be developed.

4.4.5 Complexity and Cost

The quality of the EM results not only depends on the quality of the input data but also on the quality of the experimental structure. In general, highly complex models should have a lower uncertainty. This means that if error-free input quantities are available, highly complex models should perform better than simplified models. In contrast, in experimental models, cumulative uncertainty increases with the increasing amount of data or number of PMs, which is clearly shown in Fig. 2.2 and Fig. 3.14. These figures can be used to indicate the quality and complexity of the EM. Therefore, optimisation of EM is necessary in order to obtain the minimum number of PMs with the minimum total uncertainty. The measure is based on an entropy function. Choice complexity is the mean uncertainty or randomness of the chosen PMs, which can be described by a function K_m in the following from:

$$K_m(pi1, pi2, ..., piMi) = -k \sum_{j=1}^{Mi} p_{ij} log p_{ij} \tag{4.4}$$

where, $PMij$ is the occurrence probability of a state j in the random PMs $i, j \in [1 \ \ PMi]$. k is a constant depending on the base of the logarithm function chosen, if $log2$ is selected, $k = 1$.

Costs are a relevant factor in considering methodologies because of the effects of low quality data on resource consuming activities. The cost of data quality is the sum of the cost of data quality assessment and improvement activities, also referred to as the cost of the data quality programme and the cost associated with poor data quality. The cost of poor quality can be reduced by implementing a more effective data quality programme, which is typically more expensive. Therefore, by increasing the cost of the data quality programme, the cost of poor data quality is reduced as shown in Fig. 3.14. Thus, the methodology uses cost as a second objective for optimizing measurement systems. The cost of a measurement system is computed to be the sum of PMs costs and the expenses related to testing equipment, such as data recording system in the case of long term monitoring system, this concept is shows in Fig. 4.3.

Because of limited computing resources, complexity, and engineering costs, EM can currently only be combined to a limited degree. This approach is evaluated based on the experienced and expert's knowledge. In this thesis, EM design methodology based on the expected identifiability metric optimises the configuration of measurement systems with respect to cost and performance criteria. Indeed, EM optimisation is performed according to criteria: cost and number of PMs. Both objectives need to be minimised. Results are presented in Fig. 4.3.

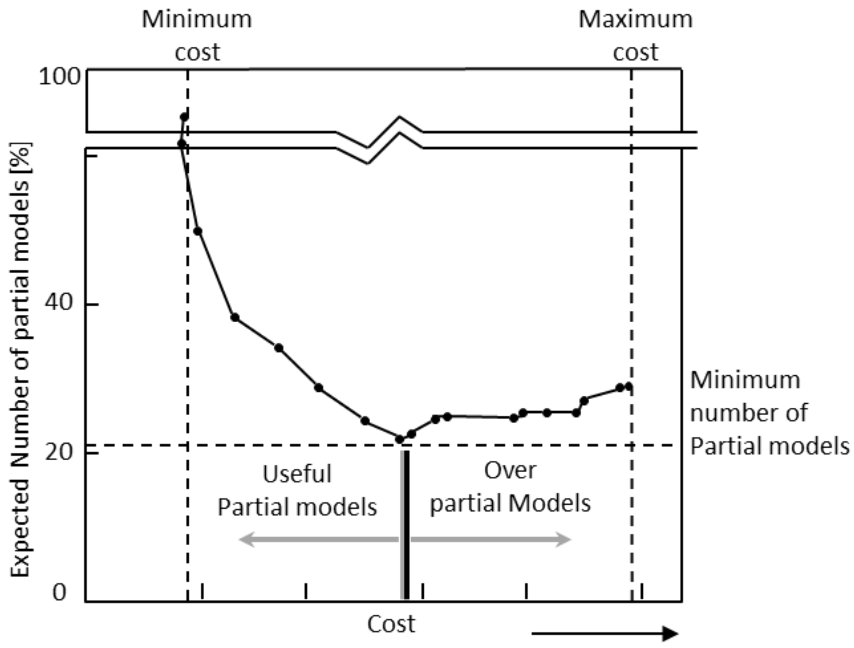


Figure 4.3: Measurement system design multi-objective optimization between number of PMs (complexity) and cost

4.5 Evaluation Criteria

Various quality measures have been used to compare the performance of the different experimental models. $logU_{E1}$ and $logU_{E10}$ are the log probabilities of selecting the EM as the best model among the all possibilities of EM. Suppose the best scoring confirmation x_i has the EM rank of Y_i in n output confirmation, then the log probability is given by:

$$logU_{E1} = log_{10}\left(\frac{Y_i}{n}\right) for\ \ logU_{E10} : Y_i = min[Yi, ..., Y_{10}]. \qquad (4.5)$$

The weighting factor for the output of EM is evaluated by input quantities over reasonable ranges for the different weighting factors. The measurement uncertainty is evaluated based on the different methods. Several alternative sensitivity analysis strategies were investigated. Measurement uncertainty using GUM, Bayesian, and MCM are compared to the different mechanical properties of EM. Different strategies of sensitivity analysis (GUM, OAT and Sobol') examine the influence of an individual quantity or part of the model on the other quantities.

Chapters 1 and 2 discussed how uncertainty can be regarded as a quantitative indication of the quality of a measurement. At first sight, this is intuitively correct: if two results of the same quantity are available, the one having a smaller uncertainty will be better than the other will. However, the uncertainty itself reveals nothing about the care put into modelling the measurand, performing the actual measurements, and processing the information obtained. For example, a small uncertainty may result because some important systematic effect was overlooked. Hence, the quality of a measurement can only be judged based on its uncertainty if it is certain that every effort has been taken to correctly evaluate it [105]. The measurement reliability is characterised by the confidence interval of measurement uncertainty. However, because the correct values are also known, both the regular and random errors can be taken into account. Hence, a safe quantity is available for the comparison of reliabilities of measuring parameters or for the optimisation of measuring models for its evaluation. Thus, the metrological analysis and – in particular, the evaluation uncertainty, reliability and robustness – can have influence the quality assurance of EM.

4.6 Comparison Experiments

Inter-laboratory studies (comparison experiments) are conducted to ensure measurement capability for commerce, evaluate national and international equivalence of measure, and validate measurement devices and measurement methods or standard materials. A common protocol employed in many inter-laboratory studies is for a reference lab to prepare materials or objects and deliver them to participating labs. The labs take measurements and report the results to the reference lab, which performs a statistical analysis. An overarching goal of many inter-laboratory studies is to establish a reference value for some measurand (the underlying quantity subject to measurement).

Using the procedures described in section 3.8, it should be possible to estimate measurement uncertainty U_E according to Eq. 3.78 associated with tensile testing. A comparison is also made using the z-score, which is explained in section 3.8.2. The z-score also depends upon the total uncertainty. In this thesis, the model quality (MQ) of model j is defined by its uncertainty and the model with the lowest uncertainty at each individual experimental model:

$$MQ_j = min(U_E). \tag{4.6}$$

The evaluation of experimental model quality is based on the total uncertainty and z-score of the individual laboratories in the mechanical properties of materials. The model with the lowest total uncertainty corresponds to the highest quality $MQ = 1$. This is assumed for the evaluation of the quality of EM in this thesis. Models with higher uncertainty have a lower quality. The measure of model quality MQ is a reference criterion in order to compare and evaluate different experimental models. The model with the highest quality is recommended. Furthermore, the total uncertainty of the experimental model for each of the models can be considered in instrumental calibration, variation in the input quantities, quality of data analysis, quality of recording software, and quality of data transformation of the scattering output mechanical properties of materials.

Chapter 5

Application

5.1 General

The evaluation method developed is considered a powerful and meaningful tool for assessing the prognostic quality of experimental models. However, this method is costly because of the extensive probabilistic studies involved. Among the research areas in civil engineering, this thesis will focus on the diverse research fields. This practice tests the boundaries of the analytical validity, illuminates the credibility of the conclusions, and estimates their robustness.

Metal, concrete, and soil are tested in opposition because of the heterogeneity of materials. Homogeneity is important because it influences the repeatability of the measurement. Repeated measurements under equal conditions will therefore lead to variation in measurement caused by material structure i.e. non-uniform grain size, arrangement, and micro cracks. In this research select three types of construction and geotechnical materials, and assumed property is presented in Tab. 5.1. It is found that the measurement uncertainty increases with the increase of the heterogeneity of the material.

Table 5.1: Materials property and measurement uncertainty

Materials	Property	Measurement uncertainty
Steel	homogeneous	2.5%
Concrete	complex and heterogeneous	3.5%
Soil	very complex and heterogeneous	10.0%

5.2 Experimental Model Quality of Tensile Test of Steel

5.2.1 Introduction

Together with hardness and Charpy impact testing, tensile testing is one of the most important quality control assessment methods used for product release certification of virtually all metallic materials. Tensile data are also extensively used in the design of products as well as for assessing the life of a component life. Knowledge of the reliability and precision of the test method is important. Measurement standards are now also required to include an assessment of the measurement uncertainty according to the GUM [6]and the JCGM [7].

This chapter discusses methods for evaluating the measurement of tensile mechanical properties and their respective uncertainties. The methodology has a systematic application associated with advanced metrology concepts and aims to guarantee metrological reliability of the results of the tensile properties as well as the possibility of implementation in industrial laboratories, research centres, and related tensile testing companies. Uncertainty in calculating the correct values reduces their value. Calculations indicate the quality test results.

5.2.2 Review of Testing Procedure

The tensile testing procedure presented in ISO 6892-1:2009 [111] was reviewed in conjunction with the modifications recommended by [112]. From the review, revised tensile testing procedure was drafted. Electromechanical tensile testing machine of 250 kN was calibrated for both load and displacement, and the expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor $k = 2$, which, for a normal distribution, corresponds to a coverage probability of approximately 95%. The tests were conducted at room temperature, and the crosshead speed rate($\dot{e}_{L_c}$) ranged from slow, middle and fast at ($\dot{e}_{L_c}$) = 0.00007 s^{-1}, ($\dot{e}_{L_c}$) = 0.00016 s^{-1} and ($\dot{e}_{L_c}$) = 0.00025 s^{-1}, respectively. The details are described in section 3.2, whereby the determination of measurement uncertainties according to the GUM, Bayesian method and the MCM are required.

5.2.2.1 Materials and Specimen Geometry

The samples used for this study were structural steel, S235 (IPE 360 and IPE 400 section, in longitudinal and transverse direction, as shown in Fig. 5.1) procured from European hot rolled profile, ArcelorMittal Steel. The samples were sectioned to produce the desired specimens according to Annex D of the ISO [111] using abrasive water cutting. Tab. 5.2 shows the chemical composition of the S235 steel. The nominal thickness of the samples was 8.00 mm and 8.60 mm for IPE 360 and IPE 400 steel, respectively. Specimens, the testing apparatus, and different strain measurement devices are shown in Fig. 5.3. Four groups of specimens were tested to individually compare the three strain measurement techniques. The specimen length and the gauge length were changed to reflect the procedures stated in the standard, as shown in Tab. 5.3. The four geometries, designed as specimen ID1, ID2, ID3 and ID4 are listed in Tab. 5.4 and Fig. Annex-B 5. The geometry of the specimen was measured and analysed using 3D laser scanner [113, 114] and calliper. Indeed, sources of uncertainty related to measurement object as well as measurement method were analysed. These uncertainties are considered in this section and the quantification of these uncertainties is not trivial. These uncertainties are analysed with mathematic model of 3D scanning measurement uncertainty considered in the quantification of uncertainty. Results represented in Fig. 5.2 show the deviation between the 3D scanned real part and the nominal width and thickness of samples. The Vernier calliper measures data deviation that is larger than 3D scanning. The focus of this thesis was not to quantify of all the error sources in 3D scanning and Vernier calliper but rather to develop a methodology for assessing the validity of the strain measurement by systematically accounting for the various sources of uncertainty and error, see Figs. Annex B 1-4.

Table 5.2: Chemical composition of steel S 235: According to Stahlwerk Thüringen, Arcelor-Mittal (Schneldbetrieb GmbH)

Weight Percentage (%)	Carbon C	Manganese Mn	Silicon Si	Phosphor P	Sulphur S	Aluminium Al	Nitrogen N
Max	0.20	1.60	0.55	0.025	0.024	0.069	0.005
Min	–	–	–	–	–	–	–

Table 5.3: Test procedure summary for group, strain device, and length

	Strain Devices	Sample Length (mm)	Gauge Length (mm)	Test Speed s^{-}1
ID1(IPE360, L)	Three technique	298.00	80.00	$7*10^{-5}s^{-1}; 1.6*10^{-4}s^{-1}; 2.5*10^{-4}s^{-1}$
ID2(IPE360, T)	Three technique	270.00	98.00	$7*10^{-5}s^{-1}; 1.6*10^{-4}s^{-1}; 2.5*10^{-4}s^{-1}$
ID3(IPE400, L)	Three technique	450.00	226.00	$7*10^{-5}s^{-1}; 1.6*10^{-4}s^{-1}; 2.5*10^{-4}s^{-1}$
ID4(IPE400, T)	Three technique	315.00	135.00	$7*10^{-5}s^{-1}; 1.6*10^{-4}s^{-1}; 2.5*10^{-4}s^{-1}$

During testing, in every case, the actual measured thickness of the specimens was somewhat larger than the nominal value, suggesting that the length measurement uncertainty was 0.08

Table 5.4: Gauge length, width, and thickness in reduced section.

Group	L_0 (mm)	b_0 (mm)	t_0 (mm)	L_0/b_0 (-)	b_0/t_0 (-)
ID1(IPE360, L)	80.0	20.0	8.0	4.0	2.50
ID2(IPE360, T)	80.0	20.0	8.0	4.0	2.50
ID3(IPE400, L)	200.0	40.0	8.6	5.0	4.65
ID4(IPE400, T)	120.0	40.0	8.6	3.0	4.65

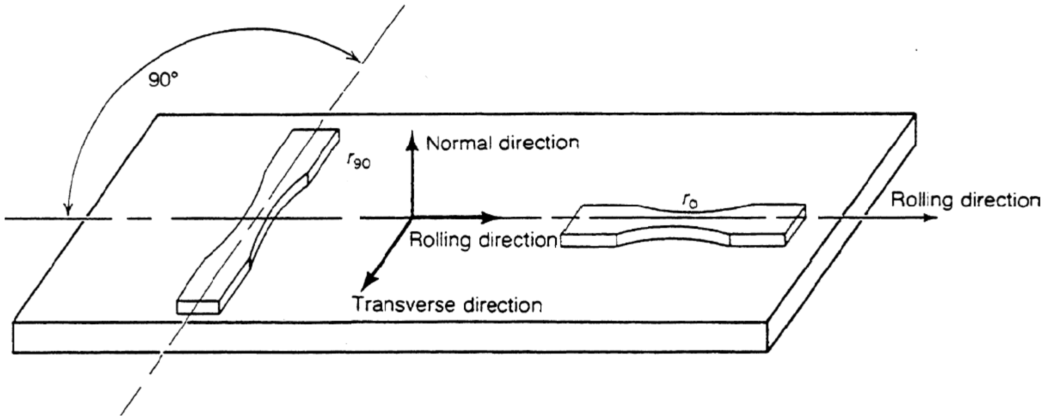


Figure 5.1: Tensile specimen orientation to determine r_0, and r_{90} in rolled sheet [115]

mm for the 3D laser scanner. Detailed description is given in Motra et al.[115, 116]. This was true to the greatest extent while producing structural steel in transverse direction, whereby the actual thickness was on mean 9.8% larger than the nominal value, as compared to an mean of 5.6% larger in longitudinal direction, for IPE360 profile. However, because the calculated values for tensile properties, presented in the following sections are taken into account for the actual thickness of the specimen, the differences between the actual and nominal dimensions

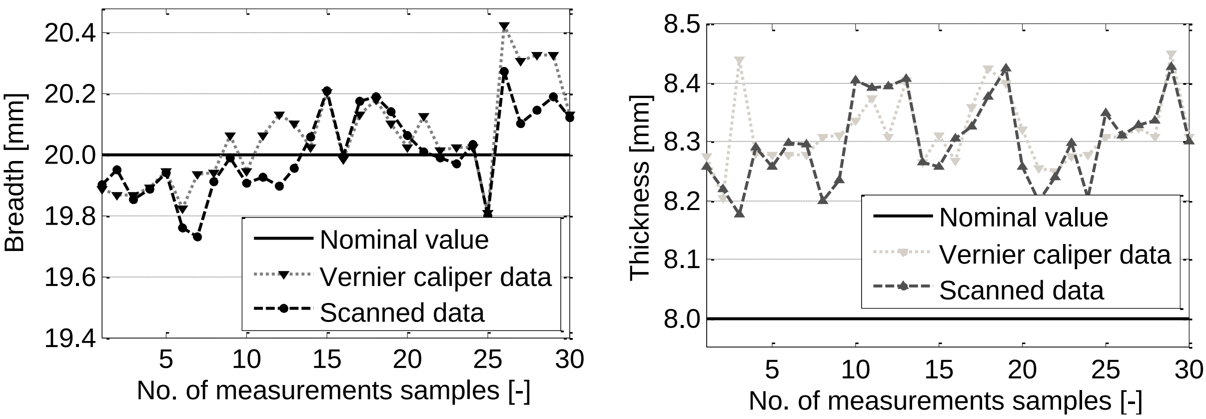


Figure 5.2: Results of the 20 mm width and 8 mm thickness gauge block scanning 30 different samples

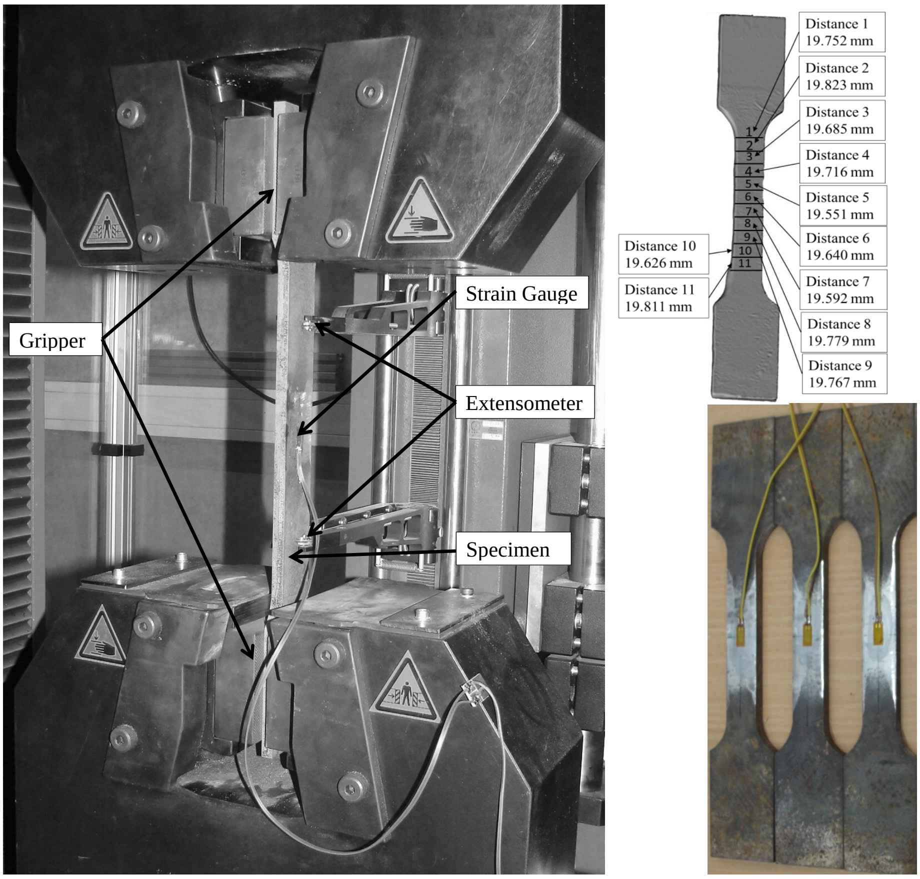


Figure 5.3: Testing apparatus for evaluating strain using three different strain-measuring devices

will not affect the results shown.

5.2.2.2 Strain Measurement

The measurement of deformation plays an important role in establishing the mechanical behaviour of materials. The two properties that are measured during a tensile test are load and displacement. The load is measured through a load cell that is installed axially in the test machine within the load path. The accuracy and reliability of displacement measurements are often in question because the magnitude of displacements is often small. Methods for measuring displacement include the tensile test, which can performed using machine crosshead motion, strain gauges, and extensometer. Machine cross head motion of electromechanical tensile testing machine of 250 kN, HBM half bridge circuit, Y series strain gauge with accuracy ± 25 $\mu\epsilon$ and extensometer of class 0.5 with uncertainty $\pm 2\%$ are used for the displacement measurement. Extensometer is highly recommended for tensile testing steel specimen. The extensometer allows for better control and achievement of the 3% turnaround point defined in the methodology and it also allows for more accurate calculation of results based on the strain compared with strain gauge and machine crosshead motion. A detailed description on the quality assessment of strain measurement technique is given in Motra et al. [115].

5.2.2.3 Residual Stresses

Similarly, residual stresses occur significantly in most structural steel members. Such stresses usually result from differential shrinkage during cooling in the manufacture of I-profiles. However, the magnitude and distribution of residual stresses in hot-rolled members depend on the type of cross section and manufacturing processes, and different patterns are proposed. If the depth of a wide flange section is large, the residual stress varies parabolically. However, it is considered constant in the web. This section investigates the influence of residual stresses on the tensile properties of structural steel. The residual stress distributions measured using X-ray diffraction techniques [117, 118] from the both structural steel specimens are shown in Fig. 5.4 and Fig. 5.5.

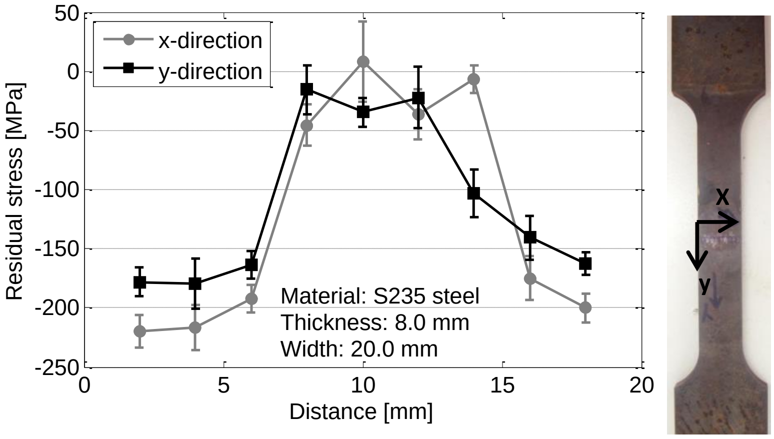


Figure 5.4: Residual stress distribution for specimen IPE360 structural steel with specimen width 20 mm

5.2.2.4 Strain Range Issues

In addition to the methods used for measuring strain in the tensile test, the strain range over which both the tensile properties are determined and the modulus fit to the stress-strain curves also influence the quality of the measurement and the calculation of the tensile properties. For many materials, the elastic part of the stress-strain curve corresponding to Young's modulus may extend only to 0.1% strain. Fig. 5.6 is a schematic representation of a stress-strain curve showing a linear region (P-Q) proceeded by a non-linear region (O-P). Modulus values tend to fall with increasing strain range, particularly if analysis methods based on fitting a straight line between discrete points on the curve are used, as one of these might occur in a region beyond

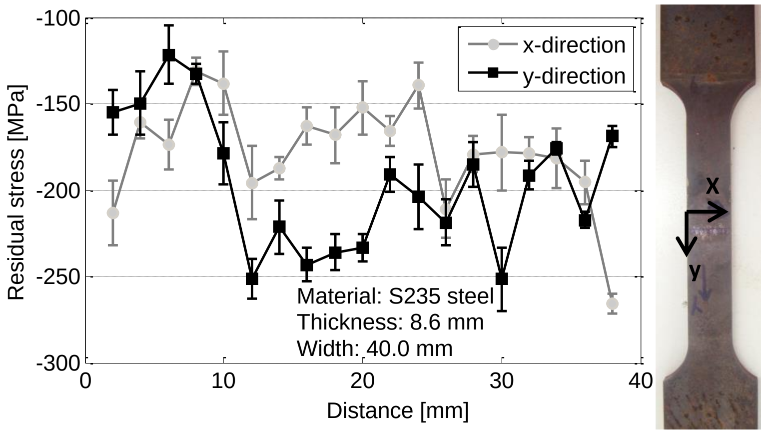


Figure 5.5: Residual stress distribution for IPE400 structural steel width specimen width 40 mm

the proportional limit.

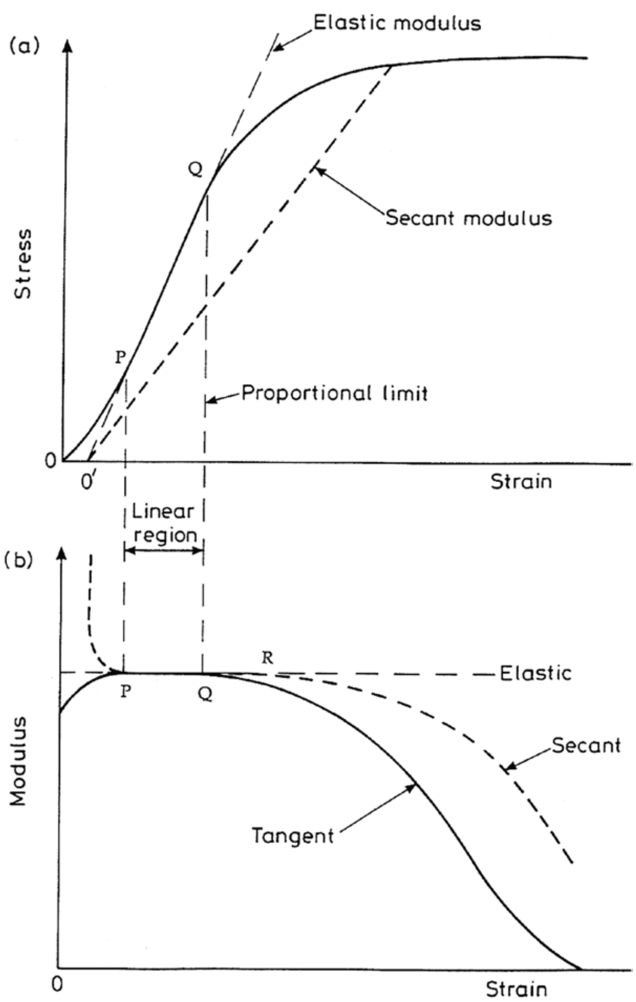


Figure 5.6: Schematic of the modulus fit to the stress-strain curve [122].

5.2.2.5 Misalignment and Bending

Misalignment and bending of specimen in the test machine is a major contributor to uncertainty and scatter in the tensile properties, and all efforts should be taken to minimize its effect by ensuring that the test machine, loading cell, and fixtures are well aligned. off-axis loading, is highly detrimental to accurate measurement. The effects are quantifiable if the misalignment is known, together with the specimen dimension. These are worse in the case of thin specimen. Alignment can be affected by the form of the specimen end-fixing. Detailed description is given in [115, 116].

5.2.2.6 Data Analysis Techniques

Although the concept of fitting a straight line to the linear part of the stress-strain curve is simple, in reality, a number of factors affect the modulus value calculated, including the noise and variability in the quality of the data, the linearity of the stress-strain curve itself, and the choice of procedures for carrying out the data fitting. In addition, data analysis software has become increasingly important in modern dimensional measurement systems, such as 3D scanners, vision systems, theodolites, photogrammetry, and coordinate measuring machines. Software computations to convert raw data to reported results can be a major source of error in a measurement system. Detailed description is given in Motra et al. [115, 116].

5.2.2.7 Reference Materials and Procedure

The use of a certified reference material as a quality check is recommended, but there is an issue because no recognised material is available for the validating tensile properties from the tensile test. Because of the mechanical testing characteristic, international cooperation to develop a primary reference procedure with strict specifications and control over the testing conditions is essential for reducing the uncertainty of the overall test procedure and improving the comparability of the data. For the primary reference materials and procedure, the main objective is obtaining test results with lower uncertainty.

5.2.2.8 Influence of Partial Models

In general, a proper measurement model for estimating the values of measurands subject to indirect measurement needs to be established. The model should provide a realistic picture of the physics involved in the measurement. The model may be either simple or complex.

For example, the modulus of elasticity (E) of a structural steel is usually determined according to the model:

$$E = \frac{\sigma}{\epsilon}, \tag{5.1}$$

where σ stands for stress and ϵ for strain. The stress is then:

$$\sigma = \frac{F}{A} \tag{5.2}$$

where A is the cross section of the specimen, F is force which is measured directly with a transducer such as load cell and the strain is

$$\epsilon = \frac{(l - l_0)}{l_0} \tag{5.3}$$

where l_0 is the original distance between the strain measured. The area is evaluated according to model:

$$A = bt \tag{5.4}$$

where b is the width of the specimen and t its thickness. Thus, in this example, the evaluation task has been broken down into PMs, subjected to direct measurement where F, l, l_0, b, and t, while A, σ and ϵ are intermediate quantities. Alternatively, one can combine these PMs into a single model for which the input quantities are only those directly measured. This would be:

$$E = \frac{Fl_0}{bt(l - l_0)}. \tag{5.5}$$

In this thesis, four models are analysed (Model 1, Model 2, Model 3, Model 4) with three different cases (case a is the strain rate ($\dot{e}_{L_c}$) = 0.00007 s^{-1} and crosshead technique is used for strain measurement, case b is the strain rate ($\dot{e}_{L_c}$) = 0.00016 s^{-1} and strain gauge technique is used for strain measurement, case c is the strain rate ($\dot{e}_{L_c}$) = 0.00025 s^{-1} and extensometer technique is used for strain measurement) and combination of other PMs.

- The measurement model of L = 298 mm, b = 20 mm, t = 8 mm, gripped ends 80 mm, three different testing speed, longitudinal specimen orientation and strain measurement used all three techniques, IPE360 steel (Model 1).

- The measurement model of L = 270 mm, b = 20 mm, t = 8 mm, gripped ends 66 mm, three different testing speed, transverse specimen orientation and strain measurement used all three techniques, IPE360 steel (Model 2).

- The measurement model of L = 450 mm, b = 40 mm, t = 8.6 mm, gripped ends 92 mm, three different testing speed, longitudinal specimen orientation and strain measurement used all three techniques, IPE400 steel (Model 3).

- The measurement model of L = 315 mm, b = 40 mm, t = 8.6 mm, gripped ends 80 mm, three different testing speed, transverse specimen orientation and strain measurement used all three techniques, IPE400 steel (Model 4).

5.2.3 Uncertainty Evaluation

A number of documents are available to help develop an understanding of uncertainty, providing guidance and advice [119, 120, 121]. The uncertainty related to each individual PMs are considered for total uncertainty calculation. Result of each PMs uncertainty discussed Motra at al. [115, 116]. There were no significant differences reported in either upper yield strength (R_{eH}), Fig. 5.7 and lower yield strength (R_{eL}). P-values for the R_{eH} were equal to 0.179, 0.213 and 0.297 for extensometer, strain gauge and crosshead techniques, respectively (n=20), as shown in Fig. 5.7. P-values for the R_{eL} were 0.198, o.321 and 0.408 for extensometer, strain gauge and machine crosshead strain, respectively (n=20). The specimen used in this experiment had consistently flat, therefore, the single point calculation was not affected by difference in strain measurement. It should be noted that for materials where these are not flat, R_{eH} and R_{eL} will be affected when using crosshead strain because of the overshoot at 4% strain and resulting overshoot at 0.2% and 2% strain values required for the R_{eH} and R_{eL} calculations.

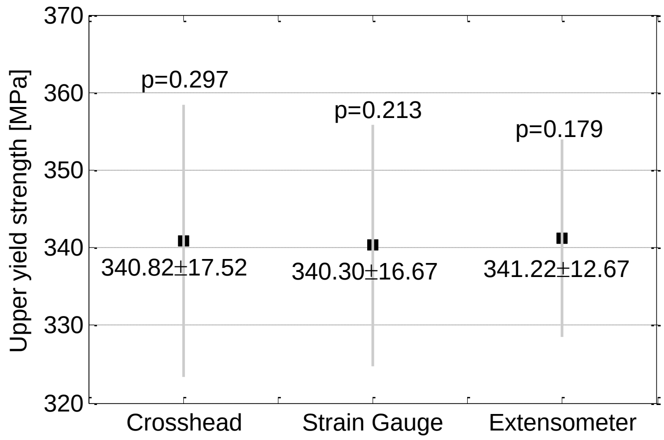


Figure 5.7: R_{eH} for three different strain measurement techniques. There was no significant difference between methods (p = 0.179 and p = 0.213 for the extensometer and strain gauge, respectively, n = 10, for Group 4 and strain rate of $7 * 10^{-5}\ s^{-1}$).

One example of PM strain measurement technique, an extensometer, is highly recommended for testing steel specimen in accordance with determined uncertainty. The extensometer allows for better control and achievement of the 3% turnaround point defined in the methodology. It also allows more accurate calculation of the results compared with strain gauge and machine crosshead motion. The bonding of strain gauges to a specimen is standard way of generating high quality and reliable strain measurements. However, applying strain gauge is time consuming and requires high levels of skill and training. Using the strain gauge technique to measure strain in the elastic region gave reliable output. However, in the plastic region, there is no more bonding with specimen, reliable strain could not be provided. The machine crosshead motion showed high variability in the strain measurement. The Young's modulus values were not in acceptable range. Therefore, the machine crosshead technique is not used for modulus measurement techniques. The extensometer technique of measuring strain could both save time and reduce costs, but the technique demonstrated relatively lower variability to the strain measurement. The development of uncertainty budgets for the mechanical properties will also help to identify particular areas of the test set-up that contribute most of the scatter and variability. The need to assure measurement quality is, therefore, a main issue to consider. The some results are presented in Annex B. The further discussion of global model quality is based on the total uncertainty.

For every output measure, a model dominates with the lowest or highest total uncertainty. The model with higher quality can be assigned by the lowest total uncertainty. Tab. 5.5 shows an example of uncertainty calculation of measurement of young modulus using strain measurement data.

Table 5.5: Example uncertainty budget for the tensile modulus test: According to the JCGM [7]

Source of uncertainty	Uncertainty	Measured value	100 x Relative uncertainty μ	Dis. Type	Divisor	100 x $\mu(E)$
Force, F	From load	calibration certificate	0.54	N	1.00	0.54
Area, A	0.041 mm^2	160.00 mm^2	1.60	R	$\sqrt{3}$	0.92
Accuracy of strain measurement	25.00 $\mu\epsilon$	1000.00 $\mu\epsilon$	1.00	R	$\sqrt{3}$	0.98
Modulus analysis method	0.85 GPa	200.00 GPa	0.80	R	$\sqrt{3}$	0.73
Repeatability of E measurement	3.00 GPa	200.00 GPa	0.95	N	1.00	1.88
N: Normal distribution			Combined standard uncertainty			2.98
R: Rectangular distribution			Expanded uncertainty ($k = 2, 95\%$)			5.97

The E-modulus, yield stress, tensile stress, and maximum tensile strain results for the IPE 360 profile and IPE 400 profile are presented and discussed in the following sections. E-modulus, which describes the elasticity of materials, is one of the most important properties in engineering design. Here, it was calculated as the slope of a stress-strain curve in the elastic region during tensile testing in the region of 0.0005% to 0.06% strain [111]. Fig. 5.8 shows the comparison of mean value of E-modulus for all specimens. As shown in Fig. 5.8, the E-Modulus extracted from the experimental stress-strain curve of IPE360 profile changes from 197.50 GPa to 205.53 GPa and of IPE400 profile from 199.10 GPa to 204.83 GPa as the longitudinal and transverse orientations, respectively. The mean of transverse specimen in this study is higher than longitudinal value, although the difference is small (2%). The coefficient of variance is found 0.024, which corresponds to a standard deviation of 4.81 GPa, which is summarized in Tab. 5.7. The specimens ID1 and ID3, which have different gauge lengths, widths and thicknesses and strain rates, are almost the same indicating that the gauge length may not have an effect on E-modulus, which is clearly shown in Fig. 5.8 and Tab. 5.7. Comparing the specimens ID1 and ID2, which have the same gauge length of 80 mm with strain rate $0.00007s^{-1}$ but different specimen orientations, it is seen that the E-modulus in longitudinal direction has 1.0% higher value than the transverse direction. Comparing the specimens ID1 and ID4, which have different gauge lengths, widths and thicknesses, the variation of mean measured E-modulus is 2.5 GPa for all strain rate, indicating the effect of varying parallel length, gauge width, grip area, and the orientation of the specimen. If the variations that occurred during machining and testing are considered, it would be reasonable to suggest that the varying geometry does not significantly affect the modulus of structural steel. Furthermore, it is seen that the measured E-modulus was more dependent on the specimen orientation of structural steel, which is also shown in Tab. 5.7.

The E-modulus measurement model quality is shown in Tab. 5.6. Analysing the measurement uncertainty, a strong influence of the testing speed and strain measurement techniques is observed. Assuming the uncorrelated input parameters, model 1 with case a shows the highest uncertainty, which was not in the acceptable range. Therefore, the machine crosshead technique for strain measurement is not used for modulus determination. Model 2 in the cases b and c shows the largest uncertainty $\approx 7.00\ GPa$. Model 1 case c shows the highest quality of total uncertainty 5.01 GPa. The quality of the further models is slightly lower. Hence, the quality of PMs is necessary in order to calculate the total model quality. In addition, in model 1 case c, the PM strain measurement and testing speed governed the lowest total uncertainty.

This section discusses the theoretical quantity and the variance of the conditional expectation that is to be estimated in order to determine an uncertainty budget associated with evaluating measurement uncertainty. First, in the case of simple measurement model (linear or linearised, monotonic as described in section 5.2.2.8, the GUM and OAT methods are easy and valid methods for calculating the contribution of all the input quantities to the variance. These can be readily implemented in any software. The GUM and OAT methods are therefore useful for performing the sensitivity index for such simple measurement models. The results show that the GUM, OAT, and Sobol' methods give estimate of E-Modulus in good agreement

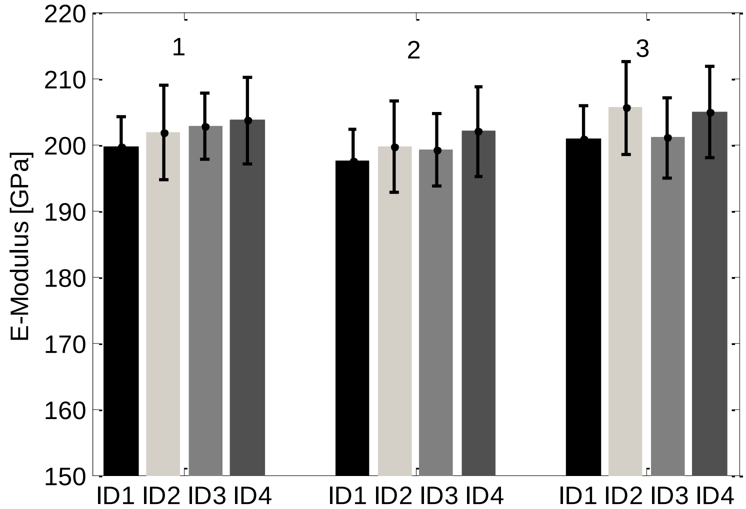


Figure 5.8: E-Modulus for all geometries (ID1, ID2, ID3, ID4) and strain rate (1, 2, and 3).

Table 5.6: Example expanded uncertainty for the tensile modulus test

	Estimates [GPa]			Total uncertainty [GPa]		
Model	Case a	Case b	Case c	Case a	Case b	Case c
Model 1	-	197.60	200.87	-	6.55	5.01
Model 2	-	199.61	205.53	-	6.85	7.01
Model 3	-	199.10	201.00	-	5.50	6.12
Model 4	-	202.00	204.83	-	6.80	6.90

Table 5.7: Summary of mechanical properties and uncertainty values: for case b

Specimen	Orientation	E-modulus GPa	Yield strength MPa	Tensile strength MPa	% Elongation at maximum force (%)	% Total extension at fracture
IPE360	Longitudinal	200.87 ± 4.36	363.40 ± 11.34	474.32 ± 12.71	19.19 ± 1.57	33.75 ± 4.40
IPE400	Longitudinal	201.00 ± 6.61	350.67 ± 15.00	438.95 ± 13.15	19.63 ± 1.82	30.18 ± 2.97
IPE360	Transverse	205.53 ± 7.10	370.34 ± 13.60	474.85 ± 13.75	18.33 ± 0.79	28.34 ± 4.08
IPE400	Transverse	204.13 ± 6.35	351.16 ± 9.64	428.61 ± 14.12	20.60 ± 0.53	34.96 ± 4.21

(Tab. 5.8).

If the measurement model is more complex or implies any interaction effect, then the GUM or OAT cannot deal with the second order effect. Then, variance based sensitivity method (Sobol') gives suitable estimation of the contribution to the variance. There are only minor differences between the three methods, as is shown in Tab. 5.8. According to the results in Tab. 5.8, force is the most influential parameter for the E-Modulus determination of steel with the assumption of uncorrelated input quantities. The interpretation of these results could lead to the conclusion that displacement and width of specimen have less contribution to the variance of the output. For simple measurement model, the GUM, OAT, and variance based methods give reliable results and can be used in measurement models.

Table 5.8: Comparison of the different SA approaches [E-Modulus determination]

Parameter	GUM [-]	OAT [-]	Sobol' [-]
Displacement	0.048	0.041	0.047
Force	0.579	0.580	0.570
Length of specimen	0.089	0.090	0.085
Width of specimen	0.059	0.058	0.062
Thickness of specimen	0.223	0.210	0.230

The test results of the five specimens for model 1, case c tested in tensile testing machine are given in Tab. 5.9. The calculation of measurement uncertainties of the test results using Eq. from 3.2 through 3.15 is shown in Tab. 5.5. The force, calliper, and 3D calibration uncertainty are taken from the calibration certificate.

The mean thickness of the test specimen using 3D laser scanner is 8.480 mm, the mean width using 3D laser scanner is 19.764 mm, the mean maximum force is 80.608 kN and the mean tensile strength is 480.919 MPa. The tensile strength and uncertainty in the tensile strength are shown in Tab. 5.9 and same way Tab. 5.5 as (480.919 ± 11.91) MPa.

Sample calculation shows that all of the sources of uncertainty including dimensions of tested specimen, applied forces, repeatability of test results, and related sensitivity coefficients can be found sequentially and added to the test results as in Tab. 5.5.

Table 5.9: Tensile test results for Model 1, case 'a' specimen

Specimen No.	Thickness t, (mm)	Width b, (mm)	Cross-section area, A (mm^2)	Tensile force F, (kN)	Tensile strength (MPa)
1	8.515	19.752	168.188	84.304	501.250
2	8.439	19.873	167.712	84.044	501.120
3	8.353	19.716	163.957	74.865	456.615
4	8.582	19.851	169.210	79.940	472.430
5	8.513	19.851	169.008	79.971	473.180
$\overline{X}$	8.480	19.764	167.613	80.608	480.919

The tensile strength measurement and calculation of elongation at maximum force model output associated with total uncertainty are shown in Tab. 5.10 and Tab. 5.11. Assuming that individual uncertainty sources are uncorrelated, Model 2 case a shows the largest total uncertainty 13.60 GPa. The tensile test model has important PMs such as dimension, testing speed, and strain measurement technique. The total uncertainty varies in the range of 9.64 GPa...13.60 GPa. The tensile test model 4 case a shows the highest quality. Similarly, the quality of elongation model at maximum force is also presented in the Tab. 5.11. These results are significantly reliable than the results of measurement uncertainty was found by Lord et al. [123], Klingelhöffer et al. [124], and Bahn et al. [125]

Table 5.10: Example expanded uncertainty for the tensile strength

	Estimates [MPa]			Total uncertainty [MPa]		
Model	Case a	Case b	Case c	Case a	Case b	Case c
Model 1	459.00	470.57	480.919	12.71	12.50	11.91
Model 2	455.46	487.31	485.09	13.60	12.90	13.10
Model 3	446.13	438.73	441.93	13.15	13.00	12.60
Model 4	445.20	444.33	446.07	9.64	10.00	10.10

The tensile strength results for both structural steels are shown in Fig. 5.9. There is a little variation between the four types of test pieces for both structural steels with respect to the aspect ratio, as similar to the results reported above. The tensile strength value for the specimens taken from the longitudinal and transverse directions were 470.57 MPa and 487.31 MPa of IPE360 profile for 0.00016 s^{-1} strain rate. A detailed description has been provided by Motra et al. [105, 116]

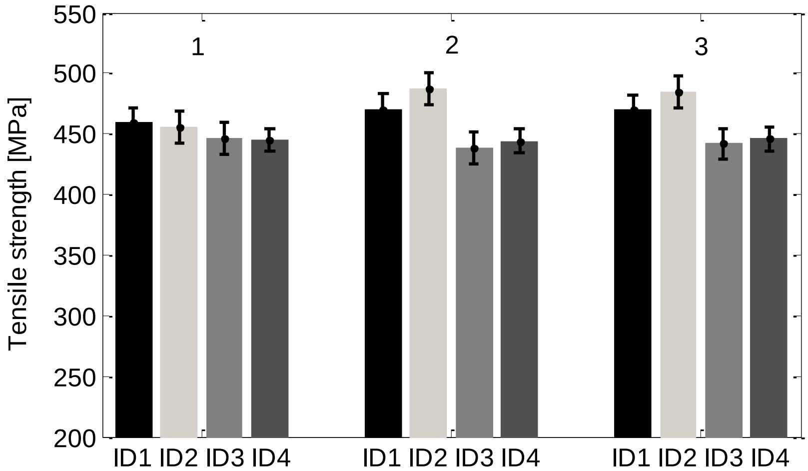


Figure 5.9: Tensile strength for all geometries (ID1, ID2, ID3, and ID4) and strain rate (1, 2, and 3).

Table 5.11: Example expanded uncertainty for the elongation at maximum force

	Estimates [MPa]			Total uncertainty [MPa]		
Model	Case a	Case b	Case c	Case a	Case b	Case c
Model 1	18.85	19.37	19.33	8.86	10.64	12.01
Model 2	18.47	18.31	18.25	9.32	8.43	12.39
Model 3	19.25	20.40	19.25	4.32	7.12	11.06
Model 4	20.68	19.25	20.56	2.60	6.86	10.10

The reliability of E-Modulus model 1 case c is the highest, as is shown in Tab. 5.12. Hence, E-Modulus Model 1 case c provides the more reliable E-Modulus value than other models. In general, engineers do not believe that a single reliability coefficient should be used for method comparison studies. If the reliability of two methods is to be compared, the reliability of each method should be estimated separately by making at least two measurements on each subject with each measurement method.

Table 5.12: Example reliability for the tensile E-Modulus test

	Estimates [GPa]			Reliability [-]		
Model	Case a	Case b	Case c	Case a	Case b	Case c
Model 1	-	197.60	200.87	-	0.85	0.95
Model 2	-	199.61	205.53	-	0.83	0.91
Model 3	-	199.10	201.00	-	0.89	0.89
Model 4	-	202.00	204.83	-	0.88	0.90

5.2.4 Uncertainty Evaluation using Bayesian method

The results presented above are based on a sample size of 60. This is a relatively small sample and certainly induces (statistical) uncertainty. In this section, the effect of sample size is analysed. Accurate values of modulus are also necessary for obtaining reliable values for proof stress, because inaccuracies in the slope or E-Modulus fit can give significant errors in proof stress, particularly if the material has a high work hardening rate in the early stages of yield. In order to evaluate the uncertainty in this estimate, or, equivalently, to assess the goodness of the available sample size, the Bayesian paradigm will be adopted. This approach has been widely accepted as the most appropriate to deal with statistical uncertainty [29, 32, 33, 34, 35, 36].

Because it was assumed that E-Modulus follows a normal distribution i.e. $E \sim N(\mu, \sigma)$, where both are random variable, an estimate of E was computed using the following equation:

$$E = \mu - 1.645\sigma. \tag{5.6}$$

Using non-informative priors, the parameter μ is t-distributed and σ^2 follows a gamma distribution. Using those distributions, a sample of E-Modulus was generated using Monte Carlo simulation from which the mean and the standard deviation were computed. The mean of E is 201.51 GPa and the standard deviation is 4.83 GPa, which yield a relative error of 2.40%. Because this is smaller uncertainty than GUM method, it can be concluded that the estimate E can be considered close to the reference value, or that the small size can be regarded as good enough for the purpose of mechanical properties estimation. In Fig. 5.10 the probability densities for the mean of statistical model of E-Modulus are presented. The posterior density, calculated with Bayesian paradigm, is then orientated closer to the likelihood with a slightly higher density.

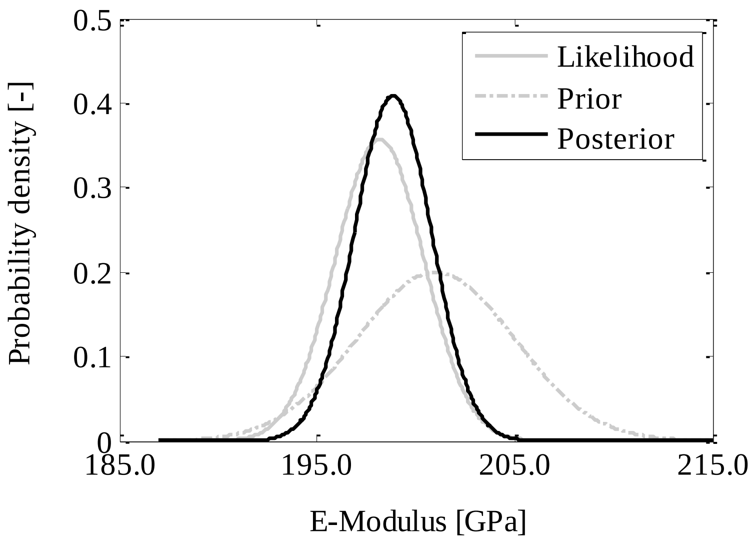


Figure 5.10: Bayesian probabilistic model for E-modulus.

Consider the same example again where $x_e = \bar{x} = 201.51$ GPa, $s^2 = 25.75$ GPa and $r^2 = 1.167$. The measurement uncertainty (Type A) according to GUM gives $\mu_\xi = 1.36$ GPa. The Bayesian method produces instead $\mu_\xi = 1.41$ GPa. Fig. 5.11 depicts the pdf Eq. 3.23, for the values in this example.

Tab. 5.13 shows the measured modulus x (model 1 with case a, b, c), the standard uncertainties $\mu_c(x)$ and the expanded (total) uncertainties $U_{95}(x)$ for 95% coverage probabilities. The weighted means x_W and their standard uncertainties μ_{x_w} are also shown.

It may be seen that the pdfs based on the enlargements of uncertainty adequately reflect the final collective state of knowledge. These pdfs tend to peak above those points having smaller uncertainties, but the area under the curve over the other points is still substantial, as shown

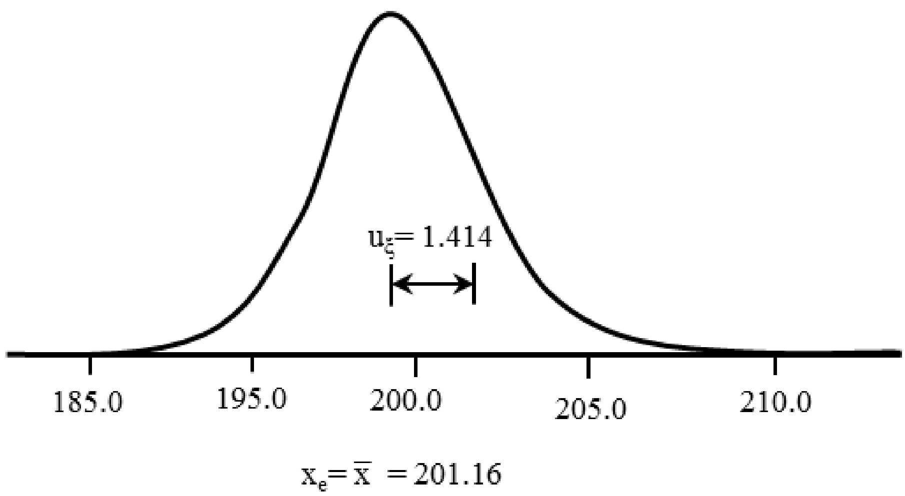


Figure 5.11: The pdf for Y with normal probability model.

Table 5.13: Measured quantity, modulus, x of the pdfs f_x, standard uncertainties $\mu(x)$ and expanded (total) uncertainties $U_{95}(x)$ for 95% coverage probabilities; also weighted means x_w and standard uncertainties μx_w.

Case	E-Modulus Estimates [GPa]	$\mu_c(x)[GPa]$	$U_{95}(x)[GPa]$	$x_w[GPa]$	$\mu_{x_w}[GPa]$
a	-	-	-	-	-
b	197.60	2.67	4.55	198.05	1.59
c	200.87	2.63	5.01	201.15	1.80

by the 95% coverage interval encompassing all estimates. In contrast, the Gaussian pdfs with weighted mean measured are much narrower, so that coverage intervals based on them may unreasonably exclude some of the extreme results. It may be checked that in all three cases the data goodness fit is calculated by the chi-square test proposed in Motra et al. [90, 105].

The distribution assumed for the output quantity Y is obviously normal, in line with the GUM uncertainty framework, but this assumption requires confirmation using a MCM technique, for both cases of a normal or a t-distribution input uncertainty associated with the input quantity X, as illustrated in Fig. 5.12. The influence on the output distribution is apparent, and it clearly illustrates one of the advantages of the MCM.

Tab. 5.14 expressed the results obtained with GUM and MCM methods, as well as its comparison in order to validate the application of MCM. The results are presented to more than two significant figures in order to facilitate their comparison of the JCGM [7]. Column 6 to 8 of Tab. 5.14 show the results obtained by applying the validation procedure of the GUM. The validation of the GUM depends on whether or not the difference between the limits of the coverage interval is larger than the numerical tolerance δ associated with the standard uncertainty $\mu(y)$. For the applications of the MCM, the GUM is not validated. However, the endpoint difference,

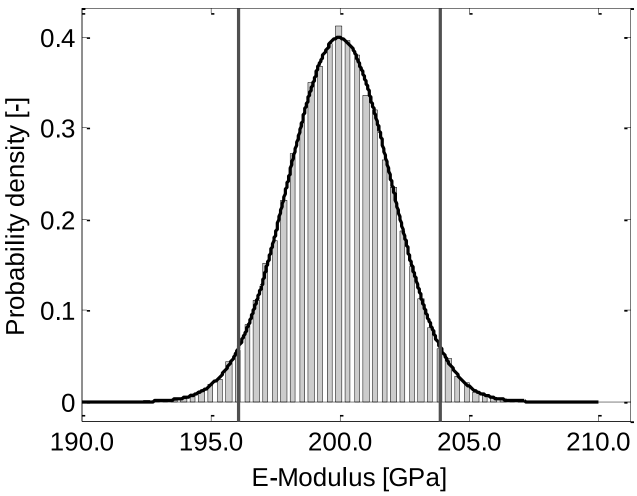


Figure 5.12: Approximate of the probability density function for the output quantity using GUM and MCM

d_{low} and d_{high}, are close the numerical tolerance $\delta = 0.05$ and for one significant digit in $\mu(\sigma)$ the validation status would be positive. Given that the coverage interval determined by the GUM is more conservative than the obtained using MCM, the minor asymmetry of the MCM result is not conclusive.

Table 5.14: Comparison of the results obtained using GUM and MCM

Method	Trail	E-Modulus	$\mu(E)$	Interval	d_{low}	d_{high}	GUM validation
	M	GPa	GPa				$\delta = 0.05$
GUM	-	201.00	9.00	[200.00 – 218.00]	-	-	-
MCM	0.35 x 10^6	201.83	7.88	[193.95 – 209.71]	0.051	0.150	No
MCM	0.50 x 10^6	201.67	7.41	[194.26 – 209.08]	0.055	0.125	No
MCM	1.00 x 10^6	201.51	6.98	[194.53 – 208.49]	0.055	0.105	No

Results of measurement model subjected only to random effect can be evaluated within the existing traditional frameworks. The case of result produced by different observations, laboratory measurement and of quantities expressed by experimental models involving systematic effects also. In most piratical applications, the uncertainty intervals obtained from existing traditional frameworks using either theory may be similar, but their interpretation is completely different. The probability distribution is obtained by performing the proposed methods (GUM, Bayesian, and MCM) treats random and systematic with stochastic dominance on the uncertainty propagation, which is the combined effect on output variables of the experimental models.

5.2.5 Inter-laboratory: Comparison Experiments

Inter-laboratory studies are conducted to ensure measurement capability for commerce, evaluate national and international equivalence of measure, and validate measurement devices and measurement methods or standard materials.

5.2.5.1 Accuracy of Mechanical Properties Determination

Eighteen laboratories were selected for the proficiency testing: Eight laboratories from Germany, one from France, one From Great Britain, three from Italy, three from Austria, three from Switzerland, and one from South Korea. All related data are taken from BAM Berlin report [126]. The reference value is determined by BAM. All tests were performed according to the DIN [103]. Tab. 5.15 shows the results of homogeneity testing as mean, standard deviation, and the measurement uncertainty of all tests. Uncertainty and homogeneity were measured as described in Section 3.8.

Table 5.15: Result of homogeneity testing

Material value E-Modulus	Std. deviation for homogeneity testing	Uncertainty in material value
184.2 GPa	0.20 GPa	0.50 GPa

5.2.5.2 Results and Discussion

The expanded measurement uncertainties are determined according to the Eq. 3.78 for all selected laboratories and are shown in Fig. 5.13. The calculated measurement uncertainties range from 2.50 to 9.60 GPa, which corresponds to 1.35% to 5.21%. The median of the results is 5.30 GPa and 2.87%, respectively. It should be noted that laboratories with particularly high uncertainty usually exhibit a high deviation from the reference value. Laboratories 1, 6, 7, 11, and 16 found uncertainty higher than the acceptable limit. Therefore, these laboratories are rejected for the E-modulus calculation. Thus, for the above calculated result from all laboratories, the E-modulus is (184.20 ± 5.21) GPa (coverage factor $k = 2$, confidence level 95%). This uncertainty corresponds to 2.87%, which is technically appropriate. The evaluation of measurement uncertainty according to existing models is possible and leads to technically acceptable results. The implementation of the computing means is complex and is only partially meaningful for the laboratory practice.

The calculated z-score using Eq. 3.82 for tensile strength is depicted in Fig. 5.14. The z-score

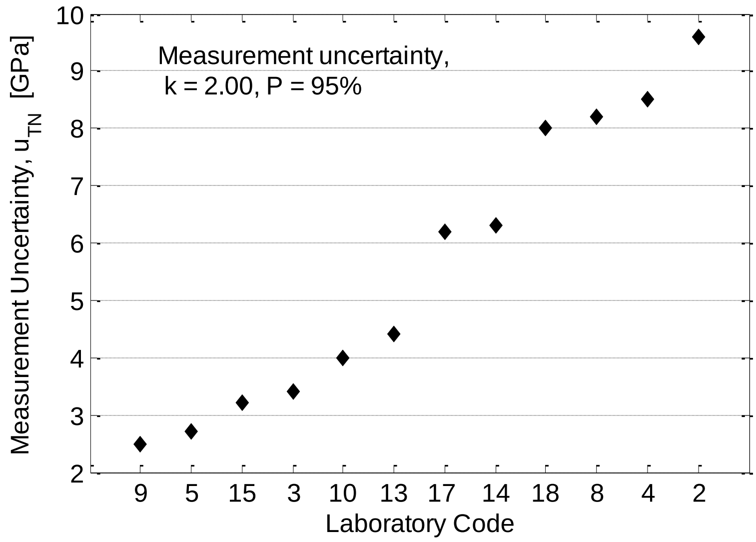


Figure 5.13: Calculated expanded measurement uncertainty for E-modulus in GPa

of laboratories 1, 6, 7, 11, and 16 have found $|z| > 3$ i.e. "unsatisfactory" results. This effect may occur because of dimension measurement error and strain measurement error. Four laboratories (3, 5, 9, and 15) have found z-score $\approx .0$ and less measurement uncertainty. Therefore, these laboratories showed higher proficiency and provided the higher quality data.

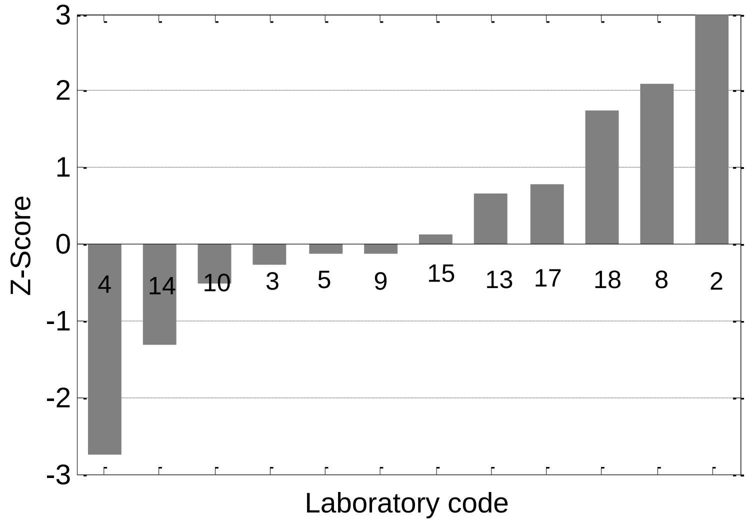


Figure 5.14: Calculated z-score for tensile strength

The proficiency testing was evaluated by the participant laboratories for the accuracy of the E-modulus and tensile strength according to the criteria of the reference value of the material. The uncertainty calculated covered both Type A and Type B according to the GUM [6] and the JCGM [7]. The reference value and uncertainty determined from the results or preliminary investigations BAM. The analysis of the results showed that the use of extensometer has resulted in no case to a successful outcome. Thus, a good performance is shown in these proficiency tests. A long-term goal of the proficiency test is the observation of the competence of the testing laboratories during a longer period. Therefore, the renewed execution of these proficiency tests is needed.

5.2.6 Conclusion

In the tensile test, there are many sources of measurement uncertainty, the influence of which should necessarily be considered even though it is difficult to quantify the effect of uncertainty sources, like strain rate, machine control modes, test piece bending, gripping methods, and so on. One of the goals of this thesis is to make an effort to reduce uncertainty. Here, the strain rate, strain measurement technique, and testing methods are the most sensitive parameters for tensile testing. The strain-rate control system is recommended for testing because it should minimise the variation of the test rates when strain-rate sensitive parameters are determined as well as minimise the measurement uncertainty of the test results. Consequently, the reduction in data scattering may not be expected because the strain rate depends on the mode of rate control, and the shape of the stress-strain curve can be influenced by the strain rate. In order to statistically analyse all individual PMs and evaluate the quality of the global model, more time and effort are required to reduce the scattering range of data. From the above example, the uncertainty, sensitivity, and reliability are clearly important properties for evaluating the quality of the global model. It is found that the measurement uncertainty is the most important model property for the proficiency testing. Establishing the robust method of proficiency testing in material properties' measurement is not an easy task, especially for developing a primary reference procedure, and cooperation between standard developer and metrologists is essential for saving time and working together effectively.

Results show that the conventional method may produce confidence levels that differ substantially from the more rigorous coverage probabilities obtained with Bayesian and MCM methods. Moreover, the latter automatically incorporates the LPU (valid only for linear models) and does not need Welch-Satterthwaite formula in connection with the doubtful concept of effective number of degree of freedom based on uncertainties. The advanced method of measurement uncertainty calculation is therefore considered the preferred means for evaluate the measurement uncertainty and the related concepts of coverage probability, coverage interval and expanded uncertainty.

A framework for the determination of measurement uncertainties, sensitivity, and reliability analysis are introduced based on the methodology described in chapter 4. A new type of quality evaluation methodology and rank of the tensile testing model is derived, based on the uncertainty and reliability in sections 4.4, 4.4, 4.4.2, 4.4.3 and 4.4.5. This methodology facilitates to use the related information of the measurement process which comprises the theoretical knowledge of the process including the physical and probabilistic nature.

Finally in this chapter was set out to establish simple, general rules, to decide upon the correct choice of method to perform uncertainty calculations. The existence of different methods for uncertainty assessment that do not always agree might be seen as an unfortunate complication by some.

5.3 Compressive Testing Model of Concrete: PCC Samples of Concrete

5.3.1 Introduction

The material properties of concrete are often a crucial factor in the structural performance of concrete elements. The quality of properties plays a major role in ensuring that the production of concrete remains stable and that the desired properties are maintained in order to assure the performance of concrete in structural engineering applications.

The testing of properties of polymer modified concrete (PCC) sample is similar to the testing of normal concrete. The metals are tested for tensile strength as opposed to PCC and normal concrete (CC). The homogeneity of metallic materials is better than in concrete samples. Homogeneity of PCC and CC is almost similar. Homogeneity is important because it influences the repeatability of measurement.

5.3.2 Laboratory Testing and Equipment Properties

Laboratory testing was performed by Flohr [127]. The mechanical properties of PCC sample of 10 cm in diameter and height of 30 cm, shown in Fig. 5.15 are measured at the age of 28, 120 days and 28, 90 days respectively. The cylinder to determine static and dynamic modulus of elasticity has a diameter of 15 cm and a height of 30 cm, which is determined according to the DIN [128]. The force should have constant increase over the time until the sample breaks and is disintegrated while axial and lateral deformations are measured simultaneously. The result is axial compressive strength i.e. stress at which the sample is disintegrated. If axial deformations are measured, Young's modulus of elasticity can be obtained. If lateral deformation is measured, the Poisson ratio of the sample is also obtained.

A large difficulty in implementing the presented measurement is sample non-homogeneity. Therefore, repeated measurement in equal condition using different samples will give a variation in measurement results caused by material properties i.e. non-uniform grain size, arrangement, testing methods and micro cracks, and measurement uncertainty calculation. Therefore, for a specified compressive strength of the concrete mixture should be proportioned for mean strength not less, on mean, and no more than 10% variation [129]. Compressive test cannot be based on only one cylinder; a minimum of two cylinders is required for each test. The importance of using accurate, properly calibrated testing devices and using proper sample preparation procedures is essential, because test results can be no more accurate than the equipment and procedures used.

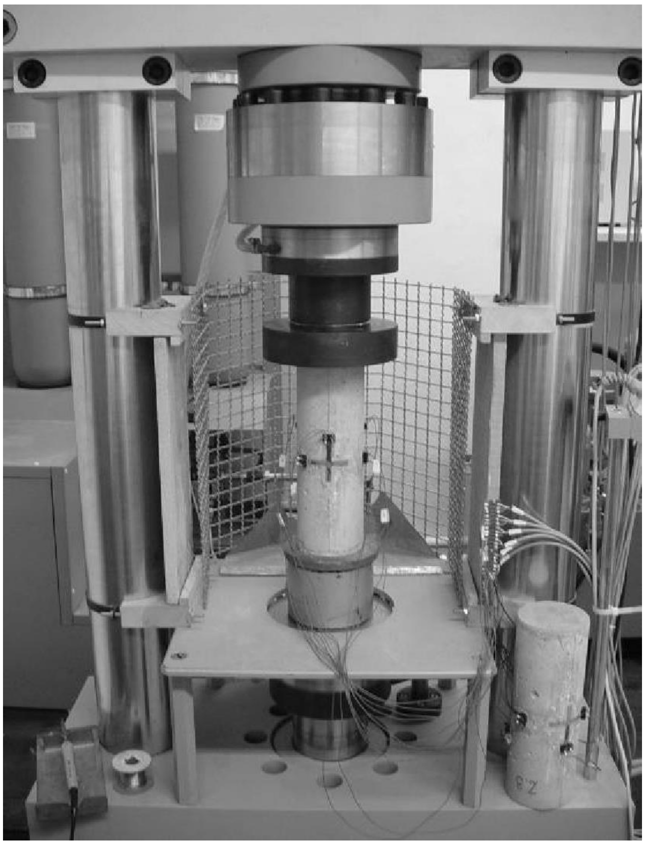


Figure 5.15: Loading device and experimental set-up of the compressive test [127]

Less variable test results do not necessarily indicate accurate test results, because a routinely applied, systematic error can provide results that are biased but uniform [129]. Laboratory equipment and procedures should be calibrated and checked periodically; testing personnel should be trained and certified at the appropriate technical level and evaluated routinely. The precision statement in the ASTM [130] indicates the in-test coefficient of variation for cylinder specimen's compressive strength made in the lab to be 2.37% and for cylinders made in the field to be 2.87%. The determination of the dynamic E-Modulus depends on Poisson's ratio which is generally not known to a high degree of accuracy; a change in Poisson's ratio from 0.16 to 0.30 reduces the computed modulus by about 11%.

Servo-Hydraulic testing Machine (SHM),is applied for testing with a maximum capacity of 630 kN using a displacement control method with a closed-loop system in order to maintain a uniform rate of loading. The permissible error is not greater than ±0.1% of the maximum load, which is given by manufacture document. An accuracy of ±0.1% is guaranteed from 20% of the load range selected to full load. Below 20% of the selected range, the maximum permissible error is 0.1% of the full load reading. The SHM has cylinder of 400 mm. SHM has both static and dynamic component load test with 0.6 kN/s loading and reloading speed and test frequency of 30 Hz.

Strain gauge of Y series of HBM products is used to measure the strain of the laboratory testing with an accuracy of 1%. The resistance accuracy is given of ±0.3%, reference temperature of 23°C, transverse sensitivity of –0.1%, the reference temperature and strain of ±1.00 $\mu m/m$, the maximum elongation of ≈ 5 $\mu m/m$, the fatigue correction is given of ≤ 30 $\mu m/m$ for $> 10^7$number of load cycles, the minimum radius of curvature, longitudinal and transverse, at reference temperature is 0.3 mm. Six strain gauges are used in each sample, in which three are attached in longitudinal and three are fixed in transverse direction.

The sample dimensions are measured using the Vernier calliper with an accuracy of ±0.02 mm, with resolution of 0.01 mm, repeatability of ±0.02 mm, with flatness ±0.005 mm, Parallelism of ±0.008 mm, and rounding accuracy of ±0.1 mm. Details of experiment and mathematical models are described in Motra at al. [90].

Characteristics of PCC comprehensive carried out by Flhor [127] and CC characteristics of concrete are studied under static and dynamic loading. Polymers (polymer-re-dispersible power with a film formation temperature of 5°C) based on styrene-acrylic ester copolymer. The mixture is made up of cements of CEM I 32.5R, 2 mm maximum grain size of sand, 2–8 mm aggregate, 8–16 mm, crushed aggregate and super-plasticizer and a de-foamer is used as additives. The mixture is according to DIN [128]. The w/c-ratio was 0.5 and p/c ratio was 0.15. Specimens were stored under 20°C and 65% humidity, in one day and packed in plastic tube up to the testing date.

5.3.3 Uncertainty Evaluation

The uncertainty related to the PCC compression test will be evaluated using the GUM method [6, 7], which is described in section 3.2. Some guidelines are available to facilitate the understanding of uncertainty in concrete [53, 131, 132]. Both the GUM and MCM methodologies are interspersed throughout the narrative with the objective of clarifying the EM and methods used as well as providing concrete illustrations of how they may be implemented.

5.3.4 Discussion of Results

The mean diameter of specimen is measured 103.36 mm. The law of propagation of uncertainty in diameter measurement is given in Tab. 5.16. Analysing the diameter measurement uncertainty, a strong influence of the standard deviation and rounding of the instrument is observed. The estimated uncertainty of sample is 0.110 mm and uncertainty contribution is 0.035 mm. Resolution and parallelism show the lower uncertainty contribution in uncertainty calculation. The contribution of flatness and parallelism of Vernier calliper to the uncertainty are given by Flack [133]. Tab. 5.16 shows the uncertainty contribution of different parameters, taking into account the sample diameter measurement. These methods are easily affected by the correlation of the measurement.

Table 5.16: Combined Uncertainty of Diameter Measurement

Source of Uncertainty	Estimation [mm]	Std. Unc. [mm]	Dis. Type [-]	Sen. Coef. [-]	Uncertainty [mm]
Std. dev.	0.110	0.035	Student t	1	0.035
Resolution	0.020	0.006	Rect.	1	0.006
Flatness	0.005	0.003	Rect.	1	0.003
Parallelism	0.008	0.005	Rect.	1	0.005
Calibration	0.020	0.012	Rect.	1	0.012
Rounding	0.100	0.029	Rect.	1	0.029
Sample Diameter	103.36			$\mu_c(d_0)$	0.047

When measuring height or length, the uncertainty components are derived from the repeatability error of the height or length. This error is a Type A uncertainty. Tab. 5.17 gives the standard uncertainty and uncertainty contribution in the sample height measurement. The standard deviation shows the strong influence of the calculation of height measurement. Other parameters (e.g. resolution and flatness) had a negligible influence.

Table 5.17: Combined Uncertainty of height Measurement

Source of Uncertainty	Estimation [mm]	Std. Unc. [mm]	Dis. Type [-]	Sen. Coef. [-]	Uncertainty [mm]
Std. dev.	3.356	0.950	Student t	1	0.950
Resolution	0.020	0.006	Rect.	1	0.006
Flatness	0.005	0.003	Rect.	1	0.003
Parallelism	0.008	0.005	Rect.	1	0.005
Calibration	0.020	0.010	Rect.	1	0.010
Rounding	0.100	0.022	Rect.	1	0.022
Sample Height	300.18			$\mu_c(h_0)$	0.950

Uncertainty in compressive strength strongly depends upon the force of the testing equipment. The uncertainties associated with the applied force include the: uncertainty of the measured load, uncertainty in reading display, uncertainty associated with eccentric of sample and uncertainty because of the angle of cap. Data for this can be obtained from research and specified by the SHM manufacture documentation for which the value may differ as the compressive strength changes. All the different types of uncertainty contribution in measurement of compressive strength of PCC, are shown in Fig. 5.16 and Tab. 5.18. From Tab. 5.18, it can be seen that the force has greater influence. GUM method (Tabs. 5.18 and Annex B.2) gives the estimate μ_c = ± 1.159 MPa. A probabilistic symmetric coverage interval for μ_c, based on a coverage factor of 1.96 (95%) of the normal probability density function was taken. The measurement uncertainty of compressive strength at 120 days obtained 3.5% and 3.23% uncertainty in tensile strength at 90 days. Details calculation steps of measurement uncertainty summarised in Tab.Annex B.2.

Figure 5.16: Uncertainties in compressive strength measurement

Table 5.18: Combined uncertainty of compressive strength measurement

Source of Uncertainty	Estimation	Std. Unc.	Dis. Type [-]	Sen. Coef. [-]	Uncertainty [MPa]
Sample Diameter Diameter	103.36 [mm]	0.35	Rect.	-8.00 [MPa/mm]	-0.28
Force	318.30 [MPa]	0.08	Rect.	14.06	1.12
Compressive strength (f_c)	30.84 [MPa]			μ_c	1.159

Uncertainty quantification of the cylinder type of polymer modified concrete (PCC) specimens is summarized in Table 5.20, Figs. 5.17 and 5.18.

The contribution of uncertainty to E-modulus, $\mu_c(E)$, is defined as the interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values could reasonably be attributed to the quantity [90]. The bar chart in Fig. 5.17 shows only the major individual components in the static E-modulus measurement of PCC. According to the Fig. 5.18, 5.17 and Tab. 5.19, measurement uncertainty of static and dynamic modulus is influenced by factors that can be internal, displacement measurement, applied force, height of sample diameter, machine diameter and sample diameter. The top five individual factors include the axial sample displacement, applied force, height of sample, and sample diameter. Uncertainty contribution for PCC sample of E-Modulus 25.775 GPa is equal to $\pm 1.44 GPa$ (i.e. 25.775±1.44 GPa).

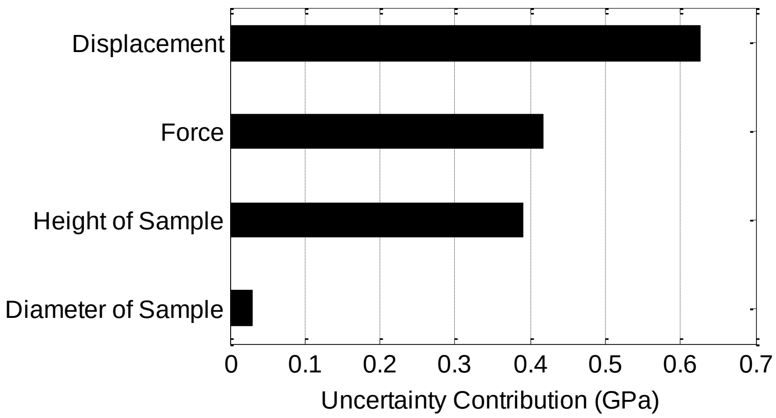


Figure 5.17: Uncertainties in E-Modulus of elasticity using GUM

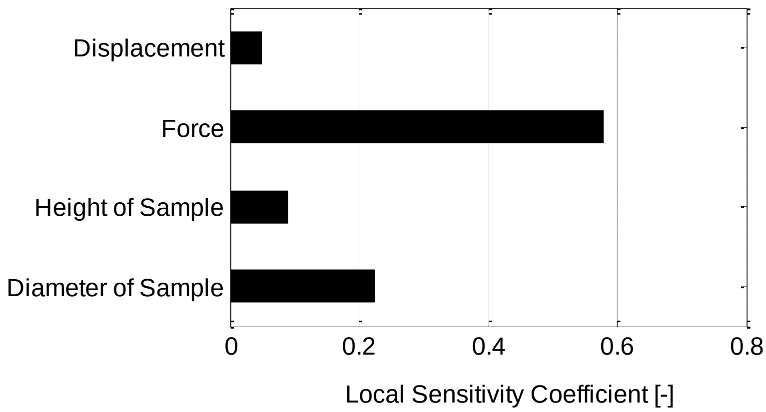


Figure 5.18: Sensitivity coefficient

The success of the evaluation of measurement uncertainties depends on the metrological problem considered, being particularly relevant to the nature of the experimental models used. Uncertainty of measurement is an essential measure of the quality of a result or testing method. Other such measures are reproducibility, repeatability, robustness, and selectivity. The validity of absolute measurement has little meaning unless the uncertainty of the equipment used to perform the experiment is known. The ability to take into account the effect of correlations depends on the knowledge of the measurement process and on the judgment of mutual dependency of the input quantities. In general, it should be kept in mind that neglecting correlation between inputs quantities can lead to an incorrect evaluation of the standard uncertainty of the quantity.

Table 5.19: Combined Uncertainty of E-Modulus of elasticity Measurement

Source of Uncertainty	Estimation	Std. Unc.	Dis. Type [-]	Sen. Coef. [-]	Uncertainty [GPa]
Displacement	0.07 mm	0.001 mm	Student t	-472.0 GPa/mm	-0.50
Height of Sample	300.10 mm	0.435 mm	Rect.	0.58 GPa/mm	0.016
Diameter of sample	103.40 mm	0.028 mm	Rect.	-2.175 GPa/mm	-0.060
Force	12.16 MPa	0.080	Rect.	5.64	0.45
Young's Modulus of Elasticity	25.77			$\mu_{(E_0)}$	0.598

Table 5.20: Results of expanded uncertainty calculation using GUM

Properties	Mean	Uncertainty	Reliability [-]
Diameter [mm]	103.36	2.35	0.82
Height [mm]	300.18	3.42	0.81
Comp. strength [MPa]	30.84	1.72	0.88
E-Modulus [GPa]	25.75	1.44	0.90
Creep [10^{-4}/[MPa]	6.67	1.26	0.82
Poisson's ratio [-]	0.25	0.015	0.84

Given the large contribution of number of samples (n) to the measured modulus and work of uncertainty quantification, especially PCC, it is interesting to discuss experimental approach to decrease uncertainty in n. It is important that each repeated measurement is independent, representative, and random. This crucial structural variable must be minimised in order to improve the quantitative measurement of PCC. The repeatability measurement uncertainty of compressive strength by GUM model is ±0.385 MPa, Young's modulus is ±0.561 GPa, Poisson's ratio is ±0.005, creep is ±0.72*10^{-4}/MPa, sample diameter is ±0.035 mm, height of sample is ±0.95 mm.

Thus, a 95% confidence interval for a normal distribution of E-Modulus of 25.75 GPa on GUM method is 24.28–27.19. This is a wide score interval, at a reliability level. The reliability index of modulus of elasticity is 0.85 and below the acceptable limit. Therefore, this variability for the measurement system is not satisfactory and adequate. It is not accepted depending upon the experimental setup, measurement method followed by the operators, or the calibration of the instruments. Therefore, a method of verification of the quality of experimental data is necessary for the mathematical modelling of physical phenomena.

The results of measurement presented in this section are used for the calculation of PCC properties. The GUM approach estimates the overall uncertainty via the law of transformation of uncertainty. Each of the components of uncertainty is established and then combined to obtain the total uncertainty. For this experiment, the main related uncertainty components are: repeatability error, force on machine, sample diameter measurement error, sample height measurement error, axial and lateral displacement error, and diameter of machine. Therefore, safety factors used ranging from 1 to 2 to achieve uncertainties that are satisfactory for this type of application. This does not mean that the measurement presented could or should have been performed with greater accuracy.

Tabs. 5.21 and 5.22 present the results of the CC and metal samples. The obtained uncertainties of compressive strength of PCC testing are 3.75%, which is not comparable with the CC data (0.3%) on Tab. 5.21. In addition, the uncertainties on compressive strength from our application are also higher than the rolled steel. This effect may occurs because of the increase of deformation of PCC sample than the CC sample. Tests on PCC have shown significant large deformation. The obtained uncertainty in E-Modulus is 2.32%, which is higher than the metal and cold rolled steel sample presented in Tab. 5.22.

Table 5.21: Strength and standard uncertainty in testing of concrete and steel

Quantity	Normal Concrete [134]	Rolled Steel [125]
Stress σ	33.00 MPa	478.60 MPa
$\mu_c(\sigma)$	0.10 MPa	2.90 MPa
$\mu_c\%(\sigma)$	0.30%	0.6%

Table 5.22: E-Modulus and standard uncertainty in tensile test

Quantity	Metal [112]	Cold Rolled Steel [135]
E-Modulus	210.00 GPa	207.00 GPa
$\mu_c(E)$	0.70 GPa	0.85 GPa
$\mu_c\%(E)$	0.33%	0.41%

An alternative, more thorough, but far simpler treatment is based entirely on MCM. This involves modelling the joint probability distribution of the input quantities, simulating a sample drawn from it, and computing the value of E-Modulus (E) for each input (displacement, force, diameter of sample, height of sample) in the sample, thus obtaining a sample from the probability distribution of the output quantity. For the presented illustration, for calculation of E-Modulus suppose that the random variables modelling the input quantities are uncorrelated and all are Gaussian distribution.

Sampling from these four (input) distributions from $M = 10^6$ of values drawn from them and computing a value of (E) for each input produce a sample size M from the probability distribution of (E). Its mean and standard deviation are $E = 25.623$ GPa and $\mu_E = 1.432$ GPa, respectively. Furthermore, these happen to coincide with the corresponding values from the foregoing approximate analysis, based on Gauss's formula. The 2.5^{th} and 97.5^{th} percentiles of these samples define a 95% probability interval for the true value of E [24.191–27.055] GPa. Fig. 5.19 shows the results of implementation of MCM evaluation of measurement uncertainty.

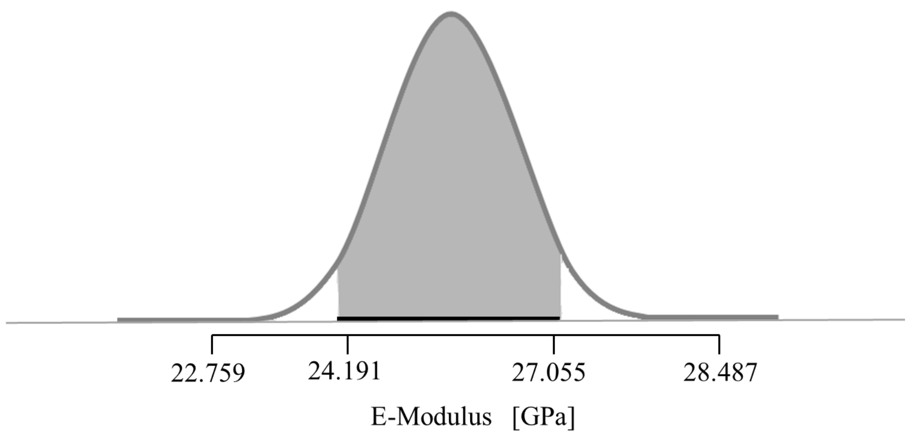


Figure 5.19: Probability density of E-Modulus of the PCC and 95% probability interval

5.3.5 Conclusion

For engineering properties of PCC measurement, measurement uncertainty is an important parameter to the measurement results. PCC sample exhibits different measurement uncertainties in engineering properties from those of CC and steel. These uncertainties come from limitation of the used instrument and transducers. The relative uncertainty contribution of measuring diameter is 0.045%, sample height is 0.31%, which are both lower than 1%. The relative uncertainty contribution of measurement of compressive strength is 3.75%, E-Modulus is 2.32%, which is higher than the CC and steel samples, which is available in literature. The relative uncertainty of Poisson's ratio is 4.0% and standard uncertainty of creep strain is 3.69%. A framework for quantifying measurement uncertainty in engineering properties of PCC using

GUM and Bayesian method is presented in the chapter above.

The MCM statistics provide helpful tools for the analysis of concrete in order to update prior information on concrete properties when additional information becomes available. In this case, formulae were given for updating the concrete properties distribution of concrete when additional test results are available. This new methodology is described in sections 4.4, 4.4, 4.4.2, 4.4.3 and 4.4.5. In addition to the specimen, the approved testing method, steady force, strain measuring system, and periodically calibrated machine reduce the total metrological uncertainty.

After this, a more sophisticated computational algorithm based on GUM and MCM statistics was developed in order to enable nonlinear regression analysis of auto-correlated data, which is of particular importance for appropriate PCC compressive models. These examples do not at all, limit the applicability of the proposed methodology for the analysis of PCC compressive testing models or other fields of engineering and non-engineering research.

5.4 Experimental Model of the Triaxial Testing of Soil

5.4.1 Introduction

Soil variability is a complex attribute that results from many disparate sources of uncertainties. The four primary sources of soil uncertainties are materials inherent variability, measurement uncertainty, transformation uncertainty and the uncertainty of the insitu and exist conditions related to the experimental scale. Measurement uncertainty is extracted from field measurements using a simple additive probabilistic model or is determined directly from comparative laboratory testing programmes. Traditionally, the geotechnical engineers deal with the uncertainties in soil properties using learn-as-you go/observation approach. However, in recent years, with more and more emphasis on reliability, the geotechnical engineering community is increasingly agreeing that any predictions must include quantifiable measures of uncertainty. The two soil strength parameters, friction angle ϕ' and the cohesion c' are most essential parameters for geotechnical ultimate limit design. Many research works showed the high variability of theses parameter [136, 137, 138]. The experimental identification of both soil strength parameters ϕ', and c' are performed using triaxial test set-up. A strategy for calibrating different constitutive soil models, efficiency, and reliability of the parameter identification have been worked out by Knabe et al. [139].

5.4.2 Test Procedure for Triaxial Compression in Soils

For the present study, isotropic consolidated and drained triaxial compression tests were performed on specimens that had been previously saturated, according to the DIN 18137 Part 2 [140]. The test specimens were obtained by compaction, for the same moisture content and compaction energy, so that same bulk density was obtained. The specimens were compacted with a diameter close to 70 mm, and a height/diameter relation close to 2 (the test standard recommends that this relation be 1.85–2.25). The isotropic consolidation is applied under three confining stresses: 38, 75, and 145 kPa. The test entails applying a constant cell pressure to the consolidated soil specimen, which is loaded to failure by moving the piston into the triaxial cell with a constant rate on the loading frame. During the testing, quantities such as horizontal (radial) stress (σ_3), pore pressure (u), axial force (F), height variation (ΔH), and volume variation (ΔV) are measured. The calibration error, output error, and operator error of measurement devices are taken into account for uncertainty evaluation. Some of the consolidation stresses were repeated to reduce experimental reproducibility. The triaxial laboratory provided automatic control of the applied cell and backpressure to an accuracy of $\pm 1\ kPa$, along with specimen volume-change measurement to an accuracy of $\pm 0.03\ ml$. The axial deformation of the test specimen was measured using a linear displacement transducer (sensitivity of $4.97*10^{-3}$ mm/mV), with the mobilised horizontal stress measured using a submersible 3 kN load cell (sensitivity of 2.4 mV/V) located inside the Perspex pressure cell. Fig. 5.20 gives an example of the engineering application of the soil test.

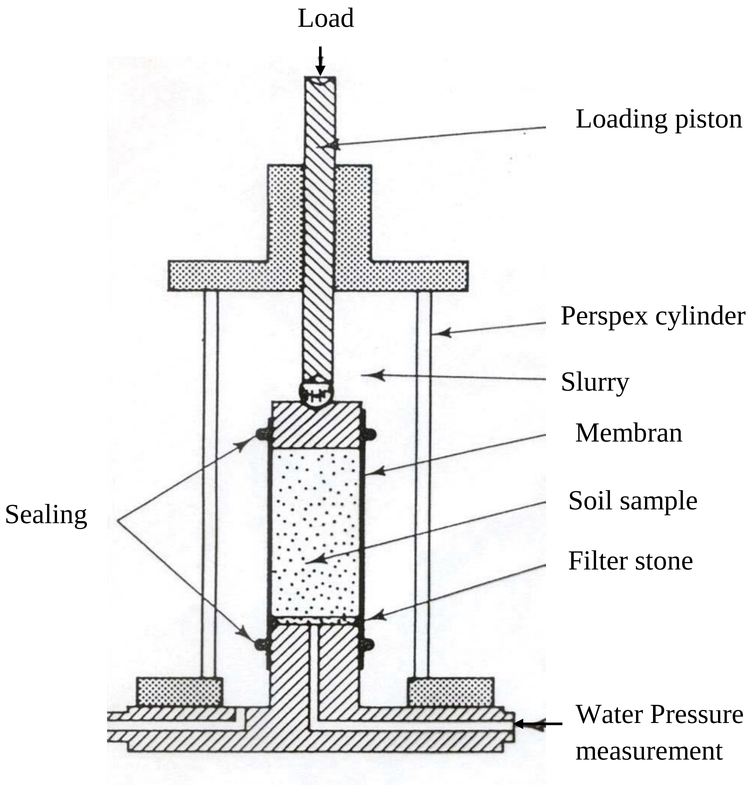


Figure 5.20: General set-up of a soil specimen inside a triaxial cell as described by the DIN [141]

Mean stress (p), mean effective stress (p'), deviator stress (q) and effective deviator stress (q') can be calculated with defined major stress, cell pressure, and pore pressure:

$$p = \frac{(\sigma_1 + \sigma_3)}{2},\ p' = \frac{\sigma_1^{'} + \sigma_3^{'}}{2},\ q = q' = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1^{'} - \sigma_3^{'}}{2} \tag{5.7}$$

where axial stress (σ_1), horizontal stress (σ_3), effective axial ($\sigma_1^{'}$), effective horizontal stress are related as above. ($\sigma_3^{'}$).

Tab. 5.23 shows the consolidation stress, mean effective stress and effective deviator stress at failure point of triaxial test. The friction angle ϕ' and the cohesion c' of given specimen can be calculated using the Ordinary Least Square (OLS) regression analysis [77], which is shown in Fig. 5.21. Regarding from the failure line on the Fig. 5.21, ϕ' = arcsin($tan\alpha$) = 37.76° and c' = a/cos(ϕ') = 13.61 kN/m^2.

By employing the common method to calculate the ϕ' and the c' magnitudes, one can

Table 5.23: Results of test for different consolidation stress

Specimen	P1	P2	P3
Consolidation stress [kPa]	38	75	145
q' at failure [kPa]	88	146	257
p' at failure [kPa]	126	221	402

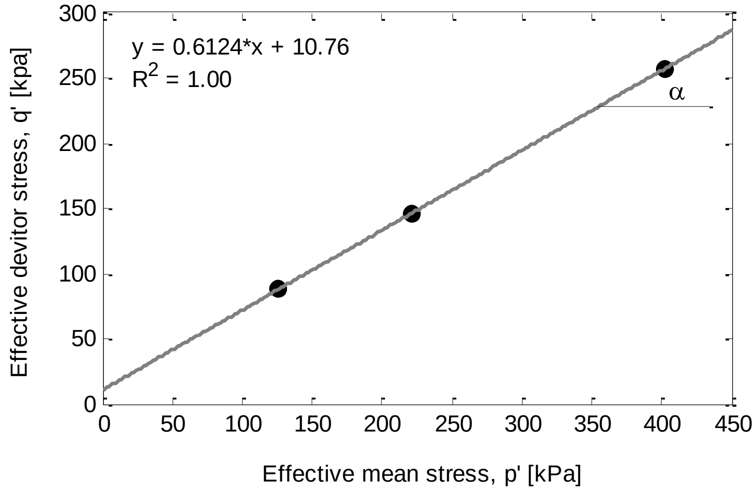


Figure 5.21: OLS regression analysis, for the evaluation of the friction angle ϕ' and the cohesion c'

identify the mean effective and effective deviator stresses acting on the failure plane. The effective axial (σ_1') and effective horizontal stress (σ_3') can be calculated with measured pore pressure (u), axial stress (σ_1) and horizontal stress (σ_3) as the following equation:

$$\sigma_1' = \sigma_1 - u, \quad \sigma_3' = \sigma_3 - u. \tag{5.8}$$

The accurate measurement of total volume and area change as well as the mean effective stress and effective deviator stress can be calculated as follows:

$$\sigma_1 = \sigma_3 + \frac{F}{A}; \tag{5.9}$$

$$q' = \frac{F}{2A}; \tag{5.10}$$

$$p' = \frac{2\sigma_3 - 2u + (F/A)}{2}; \tag{5.11}$$

$$A = \frac{V_1 - \Delta V_c - \Delta V}{H_1 - \Delta H_c - \Delta H}; \tag{5.12}$$

where F is the axial load, A is the effective area of specimen.

Table 5.24: Results after application of GUM

Specimen	$q'[kPa]$	$\mu'_q[kPa]$	$p'[kPa]$	$\mu'_p[kPa]$
1	88	2.24	126	0.43
2	146	2.22	221	0.42
3	257	2.22	402	0.42

5.4.3 The Evaluation of Uncertainty

The uncertainty related to the triaxial compression test was evaluated using the GUM [6] and the MCM [7] methods, which are described in section 3.2.

The least square estimate of the slope and intercept are 37.76° and 13.61 kN/m^2. There are at least three different ways to evaluate uncertainties associated with $\mu_{\phi'}[°]$ and $\mu_c[kPa]$: (i) least squares regression; (ii) GUM illustrated in section 3.2.1, and the GUM [6] and the JCGM [7]; (iii) MCM illustrated in section 3.2.3 and the JCGM [7]. In the following section the in this thesis will illustrate these alternatives and compare the corresponding results.

The standard measurement uncertainties using a MCM technique of $M = 10^6$ sample size, ϕ' and c' are $\mu_{\phi'} = 0.58°$ and $\mu_c = 5.01$ kPa. Similarly, from the measurement uncertainty evaluate using GUM, $\mu_{\phi'} = 0.62°$ and $\mu_c = 5.08$ kPa, such that the GUM framework can overestimate the measurement uncertainty.

5.4.4 Discussion of Results

The results of the uncertainty evaluation using both approaches are presented in Tabs. 5.24 and 5.25. Fig. 5.22 shows the PDF obtained using the MCM technique as well as the PDF obtained using the GUM with a Gaussian distribution. There are only minor differences between the two PDFs. In the expected range and toward the tails (albeit to a smaller extent), the values of the PDF obtained using the MCM are smaller than those obtained using the GUM. The PDF obtained using the MCM and the high agreement between the best estimates from GUM and MCM indicate the underlying problem. The GUM method normally used to evaluate uncertainty, which is based on the linearisation of the model function, had to be modified in order to avoid losing the uncertainty component. The standard uncertainty therefore has some slight difference. In contrast, there is high agreement between the two results.

Applying this methodology to the data from Tabs. 5.24 and 5.25 (i.e. the pairs of points (p', q') and respective uncertainties) produced the values of ϕ' and the c' shown in Tab. 5.26 and in Fig. 5.23.

Table 5.25: Results after application of MCM

Specimen	$q'[kPa]$	$\mu'_q[kPa]$	$p'[kPa]$	$\mu'_p[kPa]$
1	88	1.19	126	0.42
2	146	1.18	221	0.42
3	257	1.18	402	0.42

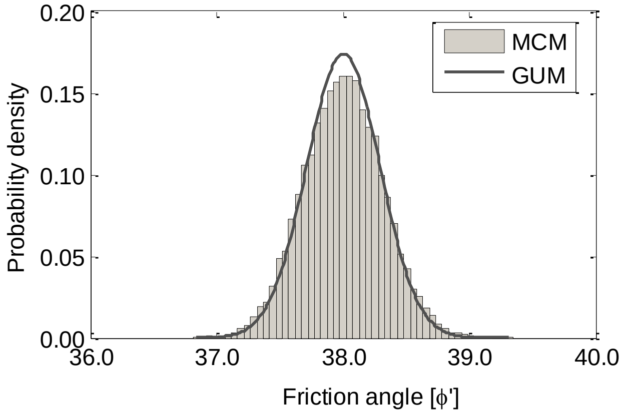


Figure 5.22: Approximation of the PDF of the output quantity ϕ' obtained using the GUM and the MCM

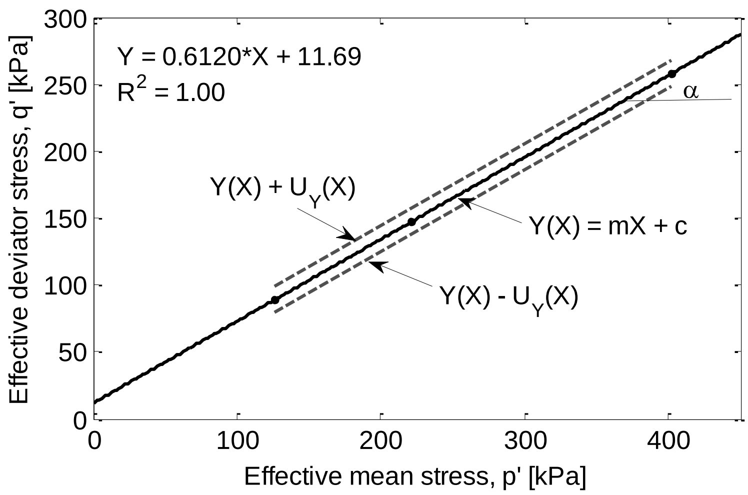


Figure 5.23: Linear regression analysis with uncertainties evaluated using MCM, for the evaluation of the friction angle ϕ' and the cohesion c'

Table 5.26: Results from the regression and MCM

—	$\phi'[°]$	$\mu_{\phi'}[°]$	$c'[kPa]$	$\mu_c[kPa]$
No Validation	37.92	—	14.03	—
After validation	37.92	0.58	14.03	5.01

Table 5.27: Results of OLS regression for specimens 1 to 3 with experimental uncertainties

—	$\phi'[°]$	$\mu_{\phi'}[°]$	$c'[kPa]$	$\mu_c[kPa]$
After validation	37.03	0.42	18.12	2.26

An important point related to the application of the least square method is that the chi-square value, which is based on the sum of the squares of residuals from the linear regression is well below the 95% quantile of the chi-squared distribution for the same degree of freedom, shown in Figs. 5.22 and 5.23. The R value was the same with different measurement uncertainties. Using this method, it is assumed that the data uncertainty matrix u_x adjusts to standard deviation of the parameter. The uncertainty values $u(x)$ and $u(N)$ were thus inflated (Eq. 3.12).

After calculating the uncertainty associated with OLS and MCM, it should be clearly and concisely documented so that it can be easily used. Fig. 5.23 and Fig. 5.24 show a set of (X, Y) data, the OLS and MCM models for that data set, and the associated uncertainty interval. The OLS regression analysis method is generally used to analyse the triaxial test data to characterise ϕ' and c'. This method was repeated with these experimental uncertainties, and the results are presented in Fig. 5.24 and Tab. 5.27. The angle of friction ϕ' and the cohesion c' for each of these three tests were calculated as well as an estimation for the experimental dispersion of ϕ' and c'.

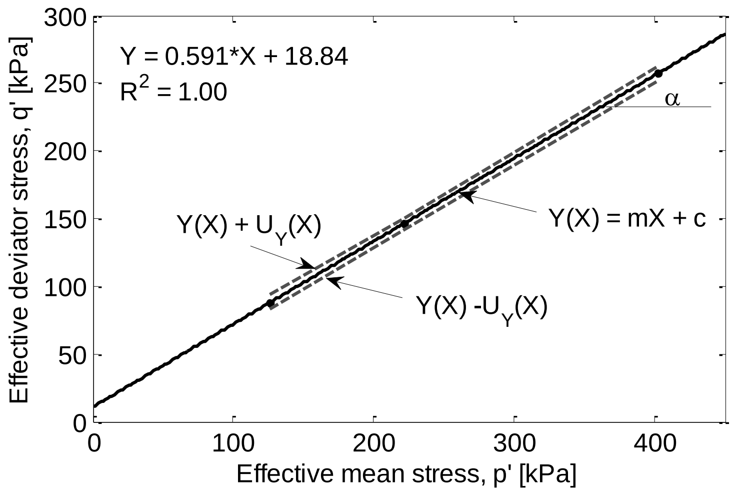


Figure 5.24: Expressing uncertainty intervals for the evaluation of the friction angle ϕ' and the cohesion c'

The larger metrological uncertainties in the linear regression model led to the acceptance of the model validation with no need for an adjustment. Without the adjustment factor, the uncertainties of ϕ' and c', resulted in lower values. For ϕ' and c', the values of direct experi-

Table 5.28: Uncertainty and reliability of ϕ' and c'

—	$\mu_{\phi'}[°]$	Reliability [-]	$\mu_c[kPa]$	Reliability [-]
Value	0.74	0.88	4.51	0.84

mental standard deviation were 0.74° and 4.51 kPa, respectively.

The analysis of the reliability of these soil strength parameters, would be high enough, as shown in Tab. 5.28. This means that the MCM approach of calculating metrological uncertainties is more reliable. Reliability parameters are highly dependent on the variation in the data quantity and are only generalised to data with similar variation. When using a certain quantity of data, reliability is clearly an indicator of the performance of an instrument.

Although linear regression with uncertainties based on experimental dispersion does result in a valid linear regression model, more replicates should be used. When obtained solely using experimental data, the uncertainties associated with the angles of friction and cohesion are smaller than those obtained by generalised distance regression with inflated covariance matrix.

The selection of an appropriate measurement for the evaluation of measurement uncertainties is, in certain circumstances, greater influence for the correctness of that evaluation with respect to the physical reality it intends to represent. The mathematical models used as the support of that representation may differ in the number of variables and its combinations, some of which are particularly common in experimental science, such as ratio like model, obervation data model and calibration model. They will all predictably introduce some degree of non linearity or asymmetry in the output quantity whose influence needs to be studied.

Table 5.29: The choice of method to the evaluation of measurement uncertainty in experiment

Model	GUM	Bayesian	MCM
Linear model (uncorrelated inputs)	✓	✓	✓
Linear model (correlated inputs)	✓	✓	✓
Non linear model (uncorrelated inputs)	X	✓	✓
Non linear model (correlated inputs)	X	✓	✓
Ratio like model	$\odot$	$\odot$	✓
Observation data model	X	✓	$\odot$
Calibration model	$\odot$	$\odot$	✓
Iterative model	X	✓	✓

where the inserted symbols refer to suitable (✓), conditional ($\odot$) and unsuitable (X) situations.

Summing up the studies of three experiemntal models of different materials related to the civil engineering, and taking into account more general classes of problem, well known and widely published, permit the construction of the following Tab. 5.29.

In this thesis, three different approaches for constructing uncertainty intervals that each have a clear probabilistic interpretation have been discussed. The solution approaches considered are capable of treating functional or probabilistic models to the degree of approximation typically required in practice. However, the modelling itself constitutes a critical stage. The choice of model dictates the solution. It is observed that, the uncertainty intervals obtained under the different methods will often be similar numerically. Even when this is the case, however, their interpretations are quite different from one another.

5.4.5 Conclusion

The use of OLS to obtain uncertainties associated with estimates of two parameters would be unreliable if there were only three points without any replicates i.e. the scarcity of data. Therefore, advanced probabilistic models account for uncertainty in soil properties by describing those properties by using PDF. The GUM and MCM are been adapted to new and improved methods for the evaluation of measurement uncertainties. OLS overestimates uncertainties and it is useful only for higher degree of freedom. For simple measurement, such as the triaxial test, both methods, GUM and MCM, give reliable results. The measurement uncertainties using GUM is slightly higher than MCM. The reason behind this is the non-linear term in the mathematical models, and assumption of rectangular distribution. Furthermore, only first and second moments (mean, standard deviation, and correlations) of GUM approach, of the distribution describing the uncertainty associated with the participating quantities are needed but results cannot be interpreted probabilistically in a useful way without any additional assumptions. Obviously, the advanced probabilistic methods might be more appropriate, especially if one wishes to recognize that one or more of these measurement uncertainties may be based on low degree of freedom.

The new proposed methodology provides consistent safety margins in design. Soil parameters show large and high variable uncertainties. GUM and MCM statistical probability approaches are rigorous ways to approach a consistent safety margin. It is believed that, the geotechnical profession should use more extensively and more routinely statistical analysis of parameters. The determination of uncertainties in triaxial soil testing model is based in sections 4.4, 4.4, and 4.4.3. The proposed methodology is highly recommended for experimental models that only have few degree of freedom. This methodology is recommended to explicitly address the variability and uncertainty of highly heterogeneous materials such as soil by implementing probability theory and analysing metrological reliability. From the above soil example was to summarize the approach taken in possible ways that take into account the variability and heterogeneity of material properties: identifying the uncertainties, gathering and classifying the available data, and then modelling these data.

In general, MCM is relatively straightforward and can be used to probabilistically simulate any problem for which the deterministic solution – either analytical or numerical – is known. However, its accuracy depends upon the number of trails (N) used in the simulation process; the Monte Carlo approach estimates their true values as the number of deterministic runs, and approaches infinity. Because of the above, accurate probabilistic simulation using MCM approach becomes extremely computational intensive.

Chapter 6

Application on Reference Object: Concrete Poles Monitoring

6.1 Measurement Uncertainties on Monitoring Models

Uncertainties are abundant in structural engineering. Major sources of these uncertainties may include uncertainties of materials properties, spatial variability, model uncertainties, and measurement uncertainties. Every measurement and monitoring have some uncertainties associated with them regardless of how carefully the measurement was made. The results of analysis based on experimental measurements also contain uncertainty. Because uncertainty affects the usefulness of measurements results and decisions that might be made using them, it is important for engineers to be able to quantify uncertainty. In fact, an inadequate consideration of experimental uncertainties can seriously compromise the final design of large, complex structures.

Because the safety factors used in design of structural strength are usually larger than those for serviceability the actual risk in loss of strength is smaller than that in serviceability. This gives rise to the need of a reliability analysis on induced structural behaviour. The application of experimental and monitoring data for a serviceability and reliability analysis exists in various fields of engineering [142, 143, 144]. The most relevant question of the application of data is: which uncertainties should be applied to the experimental and monitoring data when utilised in a reliability and serviceability analysis? An attempt is made to answer the question in this chapter. Several procedures can be adopted for the structural health monitoring during the service life, which can be based on periodical control of the structural behaviour [145]. The measurement uncertainties in strain gauge determination method is based on the GUM [6] and the JCGM [7].

The configuration of a monitoring system can be designed to be simple, using signal-measurement devices to obtain data, or it can be sophisticated in order to gather additional information. Therefore, the system of configuration mentioned here can be used in order to incorporate different measurement devices or to obtain correlations and trends of measurements. Fig. 6.1 shows a comparison of three different stages. Thereby (a) Level 1 is a simple monitoring layout to detect local and/or global maxima, (b) Level 2 is an advanced monitoring layout to detect beside local and/or global maxima also the gradient of the specific value, and (c) Level 3a and Level 3b are stochastically based on levels 1 and 2 and can be seen as structural systems if the individual measurement device is independent from each other [146]. In this chapter, strain gauge is considered as measurement device.

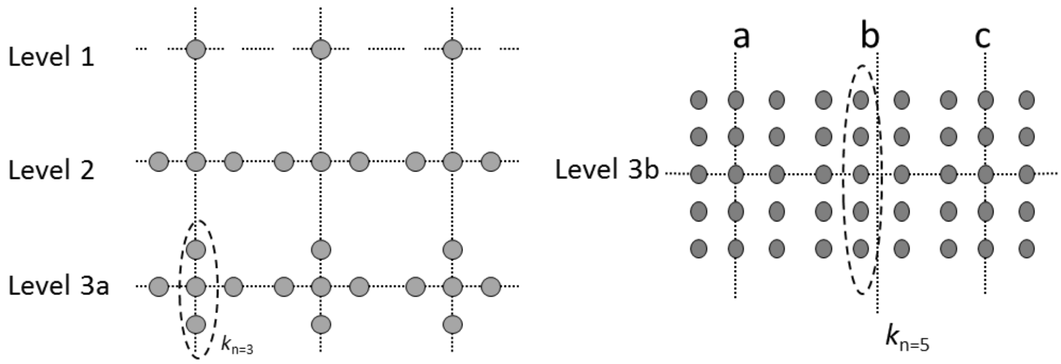


Figure 6.1: Levels of monitoring configuration [146]

Strain gauge is one of the tools most often used in strain measurement owing to their apparent accuracy, low cost, and ease of use; however, they are frequently misused, and the causes of their measurement uncertainty are badly estimated [147]. There are two reasons for measurement uncertainty: the first is due to the measurand, and the second is due to the uncertainty introduced by the measuring system. It is also important to note that systematic errors have an effect on the global accuracy of the measuring system, while random errors affect the system's precision and consequently its accuracy [148]. The quality of raw data involves the use of a model of measurement to determine the uncertainty associated with the best estimate of the value of the quality to be measured [149]. A measurement uncertainty in strain gauge according to the JCGM [7] is explained in in section 3.3.1, 3.3.2 and 3.3.3 and is used in this section.

The uncertainty inherent in measured data or measurement, uncertainty due to the positioning of the gauges, uncertainty due to the installation used to calibrate and validate model predictions are commonly acknowledged. However, these measurement uncertainties are rarely included in the evaluation of model performance [150, 151]. One reason for this omission is the lack of data on the uncertainty inherent in measured strain.

Therefore, GRK 1462 already started the installation of strain gauge during the production phase of the concrete poles in pre-stress steel and outer surface on the poles. The sensors are installed in the location Oberklobikau, high-speed Deutche Bahn project (Erfurt-Leipzig/Halle (project VDE 8.2)). The strain gauges on the tendons and reinforcing bars as well as the temperature sensors were thus applied in the concrete core. The strain gauge was fixed at the pre-stress tendon, as is shown on the right of Fig. 6.2, and the strain gauge fixed in a normal steel surface (for foundation) is shown in the middle of Fig. 6.2. Both are metallic bounded resistance strain gauges. The next strain gauge is fixed in the surface of concrete pole, which is shown on the left of Fig. 6.2. This allows an accurate determination of the strain in the pre-stressing steel as well as the determination of the temperature gradients in the concrete core over time. For the middle pole (262-27), the location of each sensor is presented in Fig. 6.3. The installation of the sensors is shown in Tab. 6.1. The positioning of the sensors was optimized by experience and expert knowledge as well as mathematically calculating model and technical conditions.

Four strain gauges of half bridge 120 ohm with k-factor 1.7, (type DMS LY-6/120) at strands and four strain gauges of half bridge 120 ohm with k-factor 2.05, (type DMS LY-6/120) at pre-stress steel were installed at 1.18 m height from foot of the pole in mast no. 262-27, which is shown on the right side of Fig. 6.3. Each four strain gauge is installed in four directions (i.e. north, east, south, and west) on each concrete pole. In addition, four strain gauge of half bridge 120 ohm with k-factor of 2.03 (type DMS VY 11-6/120) is located on the surface of foundation of concrete poles. For the prevention of strain gauge from moisture and mechanical damage is protected by rubber packing and viscous putty (AK22). These strain gauges are also installed in four directions at a height of 0.20 m from the bottom of the poles. Furthermore, four strain gauges of half bridge 120 ohm with k-factor 2.07 (type DMS LY-41/50/120) are installed in four directions. All strain gauges are produced by HBM with measurement equation described in section 3.3.1. For strain measurement, it is necessary to consider metrological uncertainty, installation, accuracy, and the special features of the strain gauge technology.

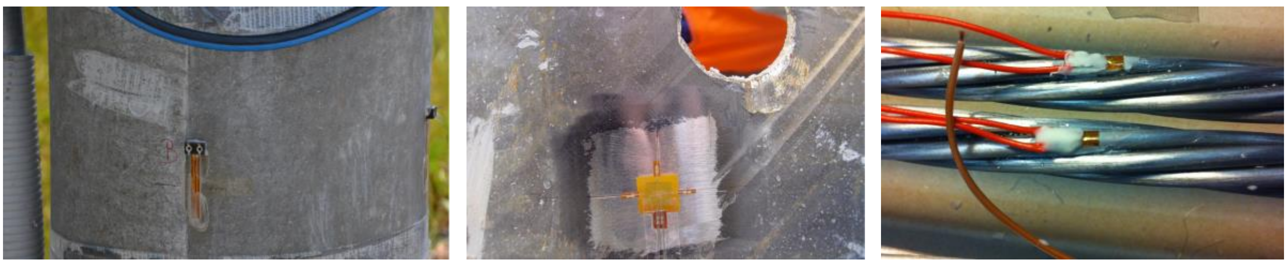


Figure 6.2: Bonded strain gauge installation in (a) left: surface of concrete; (b) middle: surface of steel in foundation and (c) right: surface of pre-stress tendon

Table 6.1: Sensor equipment for long-term monitoring

Pole no. 262-25	Pole no. 262-27	Pole no. 262-29
8 strain gauge on reinforcement	8 strain gauge on reinforcement	8 strain gauge on reinforcement
4 strain gauge on concrete surface	4 strain gauge on concrete surface	4 strain gauge on concrete surface
4 strain gauge on foundation tube	4 strain gauge on foundation tube	4 strain gauge on foundation tube
1 temperature sensor on reinforcement	1 temperature on reinforcement	1 temperature on reinforcement
1 soil moisture	1 soil moisture	1 soil moisture
	4 surface temperature	
	1 3D wind anemoter	
	1 earth pressure on foundation	
	1 wind speed sensor	
	4 accelerometer	

Furthermore, in this section, the strain measurement uncertainty is derived, based on measurement equation, described in section 3.3.1, observation equation, described in the section 3.3.2 and the posterior uncertainty using Bayesian updating method, described in section 3.3.3. Detailed description is given in section 3.3.

6.2 Assigning Uncertainties for Strain Measurements

A fundamental and new methodology for the determination of probabilistic models for measurement uncertainties has been introduced. However, the measurement data establish the realizations of the process as described by the probabilistic model leading to the question, which probabilistic model should be assigned to a measurement. The probabilistic model can be assigned to the measurement value, which equals the reference value.

The measurement uncertainties are treated similarly as the measurement equals the reference value and is then directly replaced with the established probabilistic model.

6.2.1 Probabilistic Models for the Strain Measurement Uncertainty

For the probabilistic consideration of the strain measurement problem, distribution functions are assigned to the variables which appear in Eq. 3.36, 3.37, and 3.38. The distribution functions, together with the statistical moments define the ranges for the input values, or in other words, the uncertainties of the input values (random variables) are defined. The random variables and the descriptive elements are presented in Fig. 6.4(a). As is evident in Fig. 6.4(b) a different distribution was accepted for all random variables because detailed examinations are

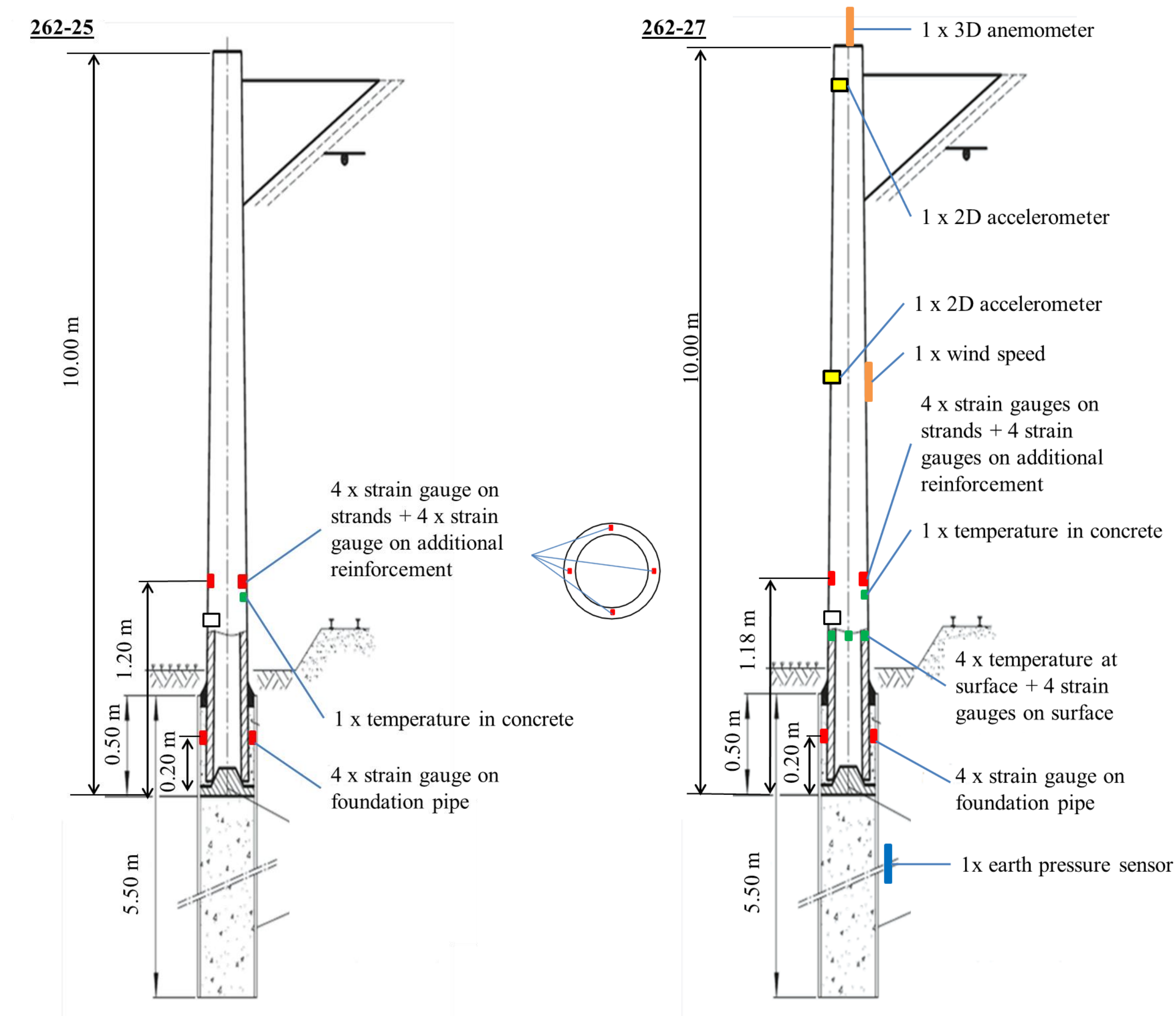


Figure 6.3: Concrete poles nos.(262-25 and 262-27) and potential sensor layout to be used for future long-term monitoring

missing regarding the distribution types. The evaluation of response function was based on the measurement equation, observation equation, and Bayesian method. The gauge factor k is assumed to be deterministic according to TML [152]. Transverse sensitivity q and Poisson's ratio of gauge calibration beam ν_0 are assumed deterministic variable because of the lack of information about these factors [153]. Moreover, the Poisson's ratio of the specimen ν is modelled according to the JCSS [154]. According to the VDI [155], the specified uncertainty is the statistical uncertainty of the mean of the strain gauge factor and a factor for the consideration of systematic effects. The factor for the systematic effects $f_{s,s}$ is there considered as a model uncertainty. The data and distribution information are taken from the VDI [155]. Following the principle of maximum entropy [6, 7] a rectangular distribution is assigned to the random variable $f_{s,s}$. The uncertainty model for the variation of the strain gauge factor $f_{s,\nu}$, temperature coefficient of the gauge factor α_k and tolerance of temperature variation curve ε_T are assumed normally distributed, as it is derived by repeated measurements [155]. The parameters of the distribution of the variation of the strain gauge factor are calculated from the distribution of the mean of the strain gauge factor assuming a 95% confidence interval and a sample size of 10 strain gauges according to the VDI [155]. The probabilistic model of the zero amplifier deviation $f_{a,z}$ and the amplifying deviation $f_{a,a}$ is assumed to follow a rectangular distribution according to the JCGM [7] utilizing the principle of maximum entropy and the product information data [84].

Model uncertainty is a stochastic measure of the model quality. Therefore, model uncertainty is determined and evaluated within the framework of uncertainty and sensitivity analysis. The model uncertainties for strain measurement $\theta_{E_{mech}}$ and $\theta_{\varepsilon_{mech}}$ are defined with the normal distribution and standard deviation 1 $\mu m/m$ in measurement equation. Assuming that individual uncertainty sources are uncorrected, the total uncertainty of strain measurement, according to GUM [6]. Furthermore, the random variables and the descriptive elements are represented in Tab. 6.2. As is evident in Tab. 6.2 a normal distribution and rectangular distribution was accepted for all random variables because detailed examinations are missing regarding the distribution types.

Table 6.2: Others source of error in strain measurement using strain sensor

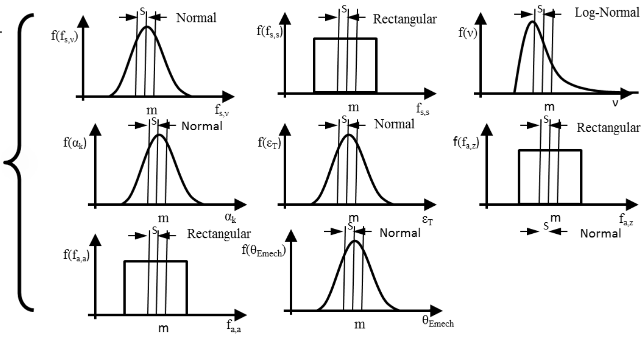
Cause of errors	Minimum	Maximum	Reference
Sensor	1.00%	3.00%	[156]
Misalignment	0.00%	3.00%	[157]
Linear deviation	0.03%	0.10%	[158]
Creep	Avoidable	Avoidable	[159]
Fatigue (cycle)	Avoidable	Avoidable	[159]
Wires of sensors	0.14%	0.30%	[156]
Data acquisition system	0.03%	0.05%	[156]
Application	0.00%	User dependent	[143]

With this method, the answer function delivers a random variable for strain. Because of there are uncertainties in the input, there are uncertainties in the result. The description of the strain is therefore both meaningful and realistic. Because our project lacked of sufficient real measurement, different reference strains under the same conditions as for the ordinary conditions were used. In this work, the temperature was kept constant at 20 degrees. However,

	Random Variable Description	Syn	Unit	Mean	Std	Dis. Type
1	Gauge factor variation	$f_{s,v}$	-	0.00	7.00x10^{-3}	N
2	Model uncertainty of gauge factor variation	$f_{s,s}$	-	0.00	4.16x10^{-4}	R
3	Specimen Poisson Ratio	v	-	3.00x10^{-1}	9.00x10^{-3}	LN
4	Temperature coefficient	α_k	%/10°C	0.10	6.99x10^{-2}	N
5	Tolerance temperature variation	ε_T	μm/m	0.00	1.19	N
6	Gauge factor	k	-	2.12	-	D
7	Transverse sensitivity	q	-	5.00x10^{-4}	-	D
8	Poisson's ratio of gauge	v_0	-	2.85x10^{-1}	-	D
9	Zero deviation	$f_{a,z}$	μm/m	0.00	6.93x10^{-1}	R
10	Amplifying deviation factor	$f_{a,a}$	-	1.00	1.73x10^{-4}	R
11	Model uncertainty	θ_{Emech}	μm/m	0.00	1.00	N

Basic Variables used for the Input
(a)

N: Normal distribution; R: Rectangular distribution;
LN: Lognormal distribution, D: Deterministic

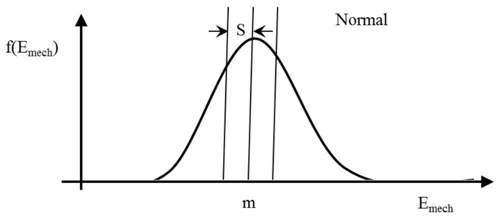


Used Distributions for the Basic Variables
(b)

$$E_{mech} = \theta_{E_{mech}} + E_{amp} + E_{app}, \qquad E_{amp} = f_{a,a} \frac{4}{k(1 + f_{s,\upsilon} + f_{s,s} + f_{s,q} + \alpha_{s,k} \Delta T_{20^0 C})} \frac{U_A}{U_B} + f_{a,z}$$

$$f_{s,q} = \frac{q(\varepsilon_q + \varepsilon_l \upsilon)}{\varepsilon_l (1 - q\upsilon)}$$

Response Function used for the probabilistic Evaluation
(c)



Total strain E_{mech} described by a Distribution
(d)

Figure 6.4: Concept of probabilistic modeling

in real project in concrete pole, temperature sensor was also installed. These uncertainties are not considered in this work and the quantification of these uncertainties is not trivial; these uncertainties will be considered in future work. Here, the methodology for the uncertainty quantification in strain measurement with the help of measured reference strain is presented. The parameters of the probabilistic model for other reference values than measured are linearly interpolated.

Influence of uncertainty quantifying in result of strain measurement is efficient if uncertainties belonging to the input quantities can be found. For this problem, there is a highly suitable tool within probabilistic theory – sensitivity analysis. This delivers the participation of the random variables, in the form of sensitivity indices in the results.

Strain distributions measured by the strain gauges directly depend on the bounded surface such as concrete, pre-stress steel and steel rods as well the direction of strain gauges location such as north, east, south, and west. The output of the strain gauges bounded on the concrete surface presented in Fig. 6.5. The mean values of strain gauge bounded on concrete surface were, 709.50 $\mu m/m$, 1080.30 $\mu m/m$, 590.02 $\mu m/m$, and 616.05 $\mu m/m$ in north, east, south and west, respectively. Furthermore, the east direction is the direction of train running. In addition, mean strain of strain gauges bounded in pre-stress steel were 1831.47 $\mu m/m$, 2645.82 $\mu m/m$, 2442.57 $\mu m/m$, and 4672.97 $\mu m/m$ in south, west, north and east directions respectively. The standard deviation of the measurement is relatively small i.e. 0.50–1.00 $\mu m/m$. Indeed, the output results for the strain measurement shows that the recorded strain of the steel rods bounded strain gauge 600.07 $\mu m/m$, 157.25 $\mu m/m$, 803.70 $\mu m/m$, and 802.30 $\mu m/m$ in south, west, north and east directions, respectively.

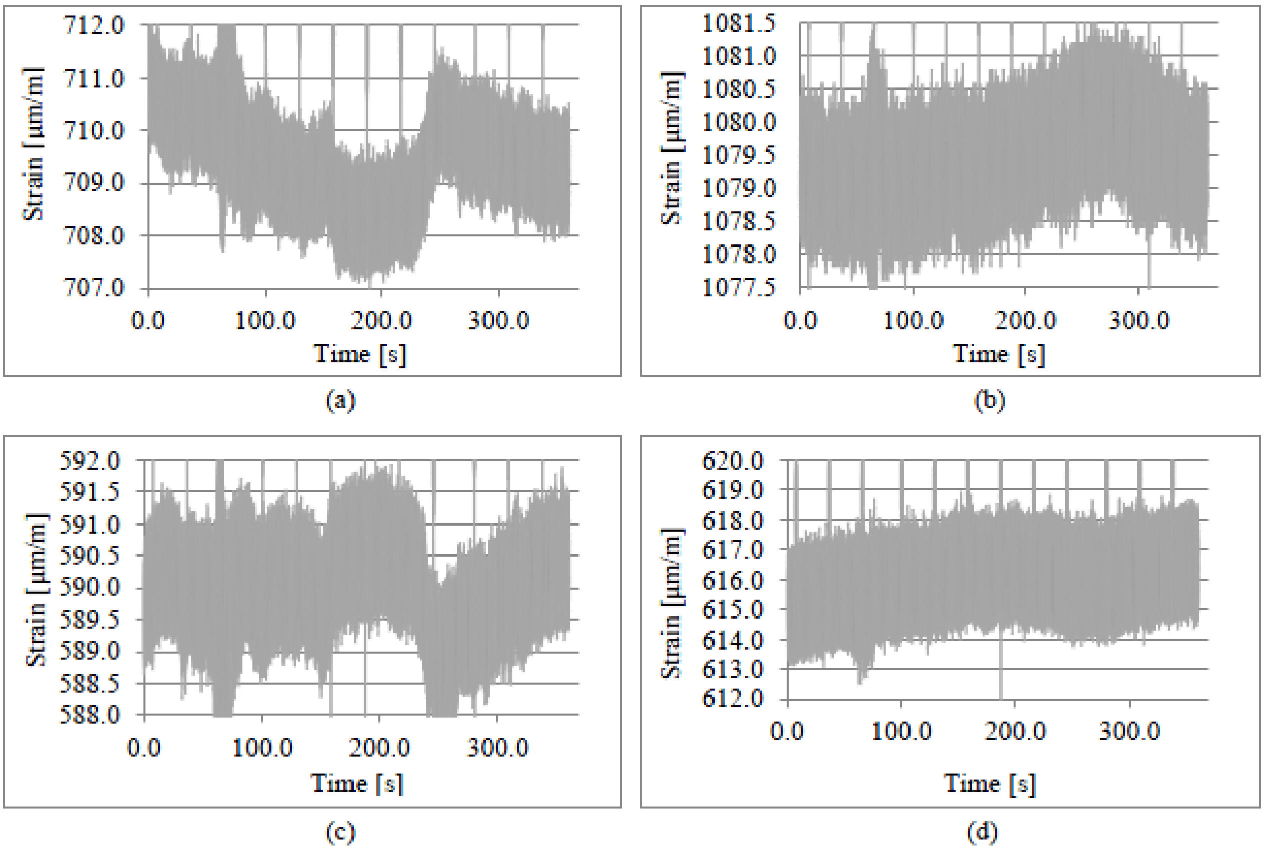


Figure 6.5: Output of the strain gauge on the concrete surface bounded: (a)North location, (b) East location, (c) South location, and (d) West location

Different quality estimation techniques have been presented with a broad range of data processing methods and it has been known that, the quality evaluation of experimental models may greatly vary depending on the data quality. Quantification of data quality is coupled with the parameter identification and experimental model quality evaluation. Descriptive statistics relies on simple statistical indicators to quantify the inherent statistical properties of data. Thus, here presents a methodology for dealing with variability and uncertainty in a strain measurement data set with advanced measurement-error-model.

6.2.2 Sensitivity Analysis (SA)

The objective of SA is to identify the critical input of metrological and observation based equations and to quantifying how input uncertainty can influence outcomes. The variance based sensitivity analysis method [40] is used to evaluate the results of the Latin Hypercube sampling method.

The sensitivity analysis results of uncertainty in strain measurement that are based on the measurement data are shown in Fig. 6.6. The most highly sensitive parameters measured by the variance-based method are model uncertainties. Mechanical strain is also the most important variable. The value of the sensitivity indices of model uncertainties and mechanical strain gradually increase with the reference strain, which indicates that an increase of these variables tends to decrease the reference strain. The value of the sensitivity indices of the uncertainty of the measurement equation model and observation based amplifier strain decreases with reference strain. The results from analysing strain sensitivity using the measurement equation are shown in Fig. 6.7. For the present problem, the most important variables are the strain gauge variation factor and the zero deviation factor. The sensitivity index of the strain gauge variation factor increases with increasing values of the reference strain. This effect may occur because an increase of the strain gauge variation factor tends to reduce the correction factor in the measurement equation model, thereby decreasing the value of the reference strain. The sensitivity index of the zero deviation factor gradually decreases with the reference strain. The model uncertainty of gauge factor variation, the correction factor of the transverse strain, and the amplifying deviation factor have a low sensitivity index, which indicates that an increase of these variables tends to decrease the value of the reference strain.

6.2.3 Strain Measurement Uncertainty on Concrete Poles

Strain measurement models in concrete poles include the familiar measurement equation, the observation equation, and the posterior uncertainty model. Like all models, measurement models are simplifications that facilitate studying the interplay between the participating quantities,

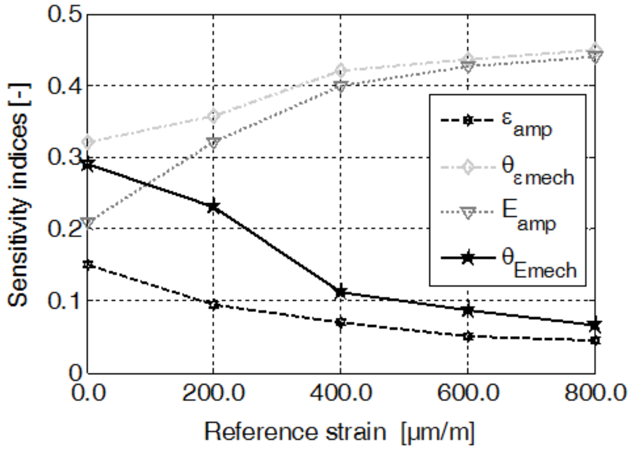


Figure 6.6: Coefficient of mechanical strain based on observation equation

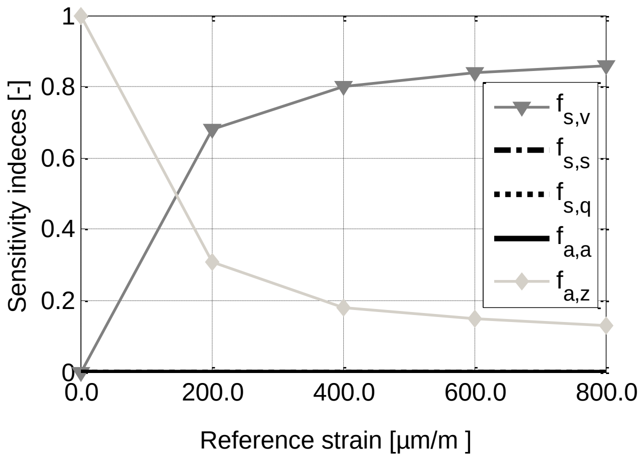


Figure 6.7: Coefficient of amplifier strain based on measurement equation

particularly those aspects that relate to the propagation of uncertainty. The standard deviation of the posterior metrological uncertainty lies between these standard uncertainties Fig. 6.8. In this section, the posterior metrological uncertainty is much closer to the metrological uncertainty based on the observation equation than the metrological equation, although these values still differ considerably. Theis is because the uncertainty in the observation is larger than the uncertainty in the metrological equation is. The systematic uncertainty in the measurement process is determined from these results. Therefore, when measuring the systematic uncertainty (instrumental error), the difference is in the PDF of the metrological and observation equations. The observation equation focusses on the relationship between the experimental data and the measurand; this relationship can be expressed as a mathematical equation that involves quantities that are representations or models of the data, the measurand, and the other quantities involved. The probability density of the measurement based equation is smaller than that of the other two methods. In many cases, the observation equation is advantageous for the metrological problem. The posterior metrological uncertainty is closer to the distribution based on the observation equation and has a slightly higher maximum probability density.

Using Bayesian updating, a PDF that reflects prior knowledge of the parameters is trans-

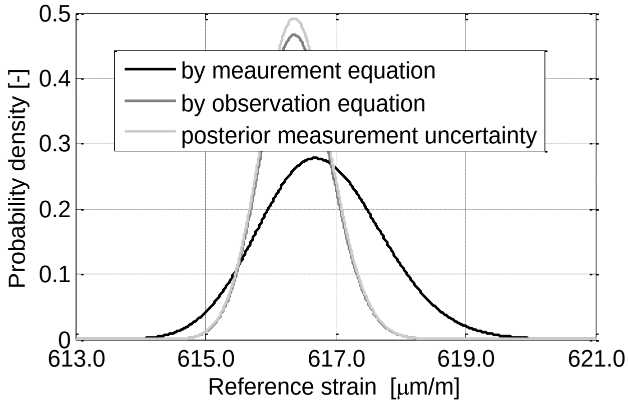


Figure 6.8: Probability densities of the measurement uncertainty, reference 616.00 $\mu m/m$

formed into a posterior PDF, which accounts for the uncertainty in the prior knowledge, the experimental data, and the predictions of the numerical model predictions. The PDFs of input quantities depend on the type of prior knowledge of each of the quantities. The likelihood depends on the type of the data that are obtained from the strain measurement. The prior knowledge model determines how the input quantities are integrated to obtain the posterior marginal PDF for the output strain. Fig. 6.9 presents the prior mean, the likelihood mean, and the posterior mean probability densities of the statistical model as well as the computed probability densities associated with the reference strain. For a reference strain with a value of 590.0 $\mu m/m$, the likelihood density of the mean is more peaked than the prior density. As can be seen, the prior density has a stronger influence on the posterior distribution than the likelihood density does. The posterior density, which is calculated with Bayesian updating, is then orientated closer to the likelihood with a slightly higher density. The influence of prior

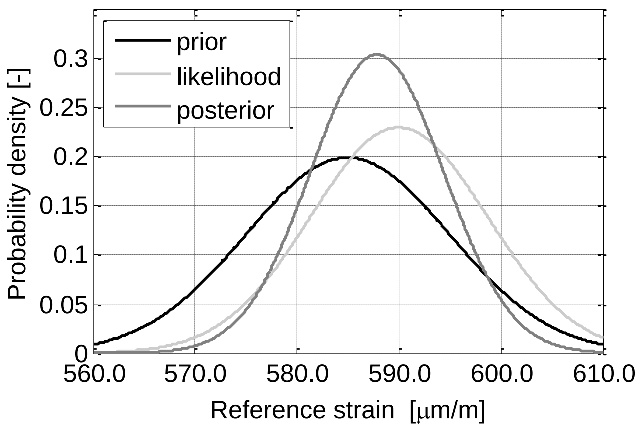


Figure 6.9: Probability densities of the measurement uncertainty mean for reference 590.0 $\mu m/m$

information on the resulting metrological uncertainty is quantitatively higher. Consequently, with an increasing number of observations, which decreases the standard deviation of the mean of the observed data, the likelihood has a significantly higher influence, and the prior density therefore loses ground. The uncertainty ranges obtained in this study vary from 88.0 to 198.0 $\mu m/m$ for the north and east locations of strain gauges bonded to concrete surfaces, respectively. These uncertainties depend on the displacement fields near the target location as well as the

6.3 Conclusions

This chapter discusses a methodology that enables the estimation of measurement uncertainties because of bonding strain gauge in the concrete pole and reinforcing steel. Furthermore, a general model based on the Bayesian updating and GUM framework is presented to calculate the measurement uncertainty of a strain gauge for strain measuring system as it offers a qualitative analysis of the principal sources of uncertainty. Different sources of uncertainty were included in the uncertainty quantification. The posterior measurement uncertainty shows higher probability density than measurement equation and observation equation. The influence of prior information on the resulting measurement uncertainty is higher. The sensitivity analysis shows that the model uncertainty factor and mechanical strain are the most important influencing factors. The proposed approach for uncertainty quantification also applies to several engineering disciplines. In general, the proposed methodology provides a fundamental framework for the determination of metrological uncertainty in strain measurement. Conventional measurement uncertainty analysis based on GUM cannot be straightforwardly applied to long-duration time series data. Therefore, in this chapter new concept introduced using Bayesian updating approach and made recommendation for best practice in interpreting monitoring datasets. There is more than one kind of uncertainty involved with measurement. It is found that, however, that the two main factor influencing the measurement uncertainty sources are the strain gauge variation factor and the zero deviation factor associated with the measurement or measurement equation.

In this section will briefly describe challenges encountered at different levels. First, the temperature sensitivity at the sensor level before and after installation, followed by the thermal behaviour of a system itself. Optimization of location and number of sensors for strain measurement, data fusion in sensors, how sensor noise is included in the measurement, sampling and quantization of the sensor output, how an estimate of the measured is recovered from the sampled, these are the major challenges.

Chapter 7

Conclusions and Future Scope

7.1 Conclusions

Within the scope of this doctoral thesis, evaluation methods for the quality assessment of experimental and monitoring models in structural engineering are developed and presented. The uncertainty in the experimental and monitoring models is determined based on the different methods. The methodology distinguishes between different probabilistic scenarios such as the uncertainty that arises from using different methods, reliability, and the complexity of the models.

The selected research framework includes a consistent methodology for the quantitative evaluation of experimental and mathematical/numerical models of structural engineering as well as their interaction, and represents a significant step towards the creation of new design principles with which the basic applicability of selected prototypical reference objects is demonstrated. Regardless of whether the models are experimental or numerical, they are not accurate i.e. they are not free of errors. All models contain some degree of error. In civil engineering, the quality of experimental and numerical models must be evaluated in order to ensure safe and robust design. The quality of numerical/mathematic models is thoroughly evaluated in Phase I of the GRK 1462 and developed framework. Here, the quality of the numerical models heavily depends on the definition and quality of the experiments. Thus, the focus of this doctoral thesis is on the development of a methodology for the quantitative assessment of the quality of the results of experiments and their exemplary implementation being an elementary component.

At the beginning of this thesis, a measurement uncertainty method was developed which is able to handle all different kinds of measurements in the field of civil engineering. This includes not only each kind of operator or function to build a connection between the different influence quantities but also the treatment of complex-valued quantities and calibration problems. The method to calculate the measurement uncertainty has to use the complete knowledge given about the input quantities, as well as the complete knowledge about the measurement function. Based on the analysis of different methods to express the state of knowledge about an input quantity, probability density functions (PDFs) are used to express all available and relevant information about the quantities. The most widely used method for uncertainty evaluation, accepted by the metrological accreditation organization, is the Guide for Uncertainties in Measurements (GUM) framework. The GUM characterizes quantities using either a normal (Gaussian) or t-distribution, which allows measurement uncertainty to be delimited by means of a coverage interval. This method is simple and straightforward but linearisation of the GUM model can lead to an inadequate representation of a system and unrealistic coverage intervals can be created. Both the Bayesian and MCM approaches treat random and systematic effects the same way for both linear and non-linear Experimental Model (EM). They are also more flexible and better adapted to practice than GUM is. For simple EM such as tensile testing model of steel, compressive testing of PCC concrete model or the triaxial testing model of soil, all methods (i.e. GUM, Bayesian and MCM) yield reliable results. The measurement uncertainty results using GUM were slightly higher than the others were, but the results of Bayesian and MCM are in good agreement at 10^6 simulation trial. When using MCM, more time and processing capacity are required to calculate metrological uncertainty in non-linear complex models. Because the accuracy of the MCM result mainly depends on the number of Monte Carlo simulations, the efficient implementation is crucial for obtaining a reliable measurement. Furthermore, it works extremely well for both simple and complex EM. For the purposes of civil engineering, all methods are application for uncertainty quantification. It is found that, MCM method is useful for the scarcity of data like triaxial soil model.

It is important to report the uncertainty. For example, in this thesis, the calculated E-Modulus uncertainty for the tensile testing of steel model was (201.00 ± 6.61) GPa using GUM method, (201.51 ± 4.83) GPa using the Bayesian method, and (201.51 ± 5.01) GPa using MCM method. For the compressive testing of the PCC concrete model, the calculated E-Modulus uncertainty was (25.75 ± 1.44) GPa using the GUM method and (25.62 ± 1.43) GPa using the MCM method. Furthermore, the uncertainty calculation for triaxial testing of soil model for cohesion was (14.03 ± 5.01) kPa and uncertainty in friction angle was found to be (37.92 ± 0.58)° using MCM. For all above calculations, the confidence level of 95%, with the note that the reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor, which provides a level of confidence of approximately 95%. Therefore, proposed methodology is work for civil engineering experimental models.

Sensitivity analysis is another way of analysing individual measurement scenario in detail determining the influence impact of a single quantity on the overall measurement uncertainty. Again, this approach is relatively new to measurement uncertainty evaluation with the MCM. In this thesis, GUM (local), OAT approaches and more advanced ones using variance-based (Sobol') methods are compared. It is now necessary to analyse results of the different approaches in practice applying more advanced real life measurement scenarios. Then it would also be interesting to implement and compare more and different approaches. All approaches fit good in material measurement testing.

In this thesis, it will be shown how reliability theory can be applied to measurement tasks. Subsequently, it was shown as the modelling process where measurement methods influence the reliability of measurement method and reliability of measuring instruments. Metrological traceability to a metrological reference such as a measurement unit, a reference procedure, and a reference material is considered fundamental in calibration and measurement. In order to confirm the reliability of a measurement result, statement of metrological traceability chain together with attributed measurement uncertainty is required for calibration and testing laboratories. The Tahuchi method of robustness is implemented for tensile steel test for selection of best combination of input parameters. In general, highly complex models should have a lower uncertainty. This means that if error-free input quantities are available, highly complex models should perform better than simplified models. Furthermore, the total uncertainty in the EM increase with the increasing the number of PMs. In this thesis, it is tried to found the optimal point, having the minimum uncertainty, minimum number of PMs and optimal cost of the EM. For this purpose, the EM are optimised with respect to the minimum uncertainty, the minimum number of PMs, and the optimal cost. It can be illustrated by the example that the tensile testing steel model shows 0.95 reliability index for E-Modulus, 0.90 for E-modulus of compressive testing of PCC concrete and 0.84 for cohesion of triaxial soil model.

This research aimed to develop methods for assessing quality and improving the EM. Uncertainty is regarded as a quantitative indication of the quality of EM. Models with lower total uncertainty have a higher quality, while model with higher uncertainty have a lower quality. When conducted in combination, uncertainty, sensitivity, reliability and complexity analysis, users of EM may be more informed about the confidence that can be placed in EM results. If the information for assessing these probabilistic factors is available, it is easier to determine the quality of the EM and thereby support a decision. For example, model 4 case a EM for the determination of tensile strength of steel is better in quality than others models based on the total uncertainty with minimum PM.

The inter-laboratory study of mechanical properties in the tensile test presents a fully developed approach to the problem of estimating the value of a measurand and characterizing the associated uncertainty. Inter-laboratory studies constitute a 'top-down' approach to uncertainty evaluation that is most likely to take into account contributions from all sources of uncertainty, including those that uncertainty budgets, in the 'bottom-up' approach of the GUM, may fail to recognize or assess properly. Based on the present study measurement uncertainty and z-scores could be suggested as the quantification of laboratory proficiency. For example, laboratory model 9 with total uncertainty $\pm 2.60\ GPa$ and z-score of –0.20 is more proficient than others for determining the E-Modulus of steel.

A framework for the determination of measurement uncertainties is introduced building upon the successor documents of the GUM. This framework, which is elaborated on the example of strain measurements, accounts explicitly for the assignment uncertainty of a probabilistic model to a measurement value and for model uncertainties.

Bayesian updating derives a new type of measurement uncertainty called the posterior measurement uncertainty. Both the prior and the likelihood are informative distributions as the prior measurement uncertainty is associated with prior knowledge about the measurement process i.e. with a measurement equation and an associated uncertainty model. The likelihood is associated with probabilistic models of observations. This approach facilitates the use of all metrological information, including the theoretical (i.e. physical and probabilistic) and empirical knowledge of the process.

The measurement uncertainties were determined and a sensitivity analysis of the involved models was performed. With the results of this sensitivity analysis, a monitoring system can be designed for a minimum measurement uncertainty as the probabilistic models of the random variables are directly linked to product specification data. The model uncertainties introduced for calculating the measurement uncertainties can have a significant sensitivity as it is the case for the observations based measurement uncertainty.

Consequently, measurement uncertainty is a property of the method i.e. of the measurement process and measurement result. Assuming that the measurement is the main source of uncertainty, metrological uncertainty can be estimated based on performance characteristics. Information is obtained via the validation of experiments. In addition to assessing the quality of EM, it is important conduct to inter-laboratory testing because this is sometimes the only way of identifying the systematic uncertainties in the EM.

This research focussed on assessing the quality and improving the approaches of EM. The approach presented in this thesis is feasible for assessing and improve the quality of EM in a continuous way. In this study was to develop a methodology for determining quality of EM concerning different materials related to civil engineering. Such methodology, aiming a guarantee the metrological reliability of the results to the EM, as well as the possibility of implementation in validation, calibration and input parameter determination of numerical/ mathematical models and industrial laboratories, research centres, in the material testing laboratories and monitoring work.

7.2 Future Scope

The proposed quality assessment of EM is based on experience, review of the existing uncertainty determination methods, sensitivity, reliability, and complexity analysis. A possible direction for future research related to assessing quality is further validated in the context of robustness and the cost of EM.

Further case studies can be evaluated by considering different types of input distribution in the EM and determining the uncertainty with respect to those distribution types and evaluating more quality-related attributes of the proposed methods for assessing quality. Additionally, methods of different probabilistic factors could be studied in more details by considering different factors, for example, software tooling, expertise, and more complex examples. Finally, the answers to these questions as well as the allowable uncertainty level have yet to be determined.

Next proposal for future work is to use measurement systems for optical measurement of deformation of the specimen during the experimental test. This type of measurement decreases the interaction between the measuring system and the specimen, speeds up preparation times, and reduces the influence of the operator on the reliability of measurement results (measurement with strain gauges requires a highly-skilled operator). Furthermore, the influence of material properties and other uncertainties in the simulation behaviour of the structural component must be investigated.

Based on the present study, the classical parametric tests should also be involved in evaluation of data of inter-laboratory comparisons with small number of participating laboratories. Further studies can be developed by using Bayesian approach to the problem of estimating the value of a measurand and characterizing the associated uncertainty. The Bayesian method of analysing inter-laboratory data suggests three modifications, one of which is more robust to outliers.

A framework for determination of uncertainty in the strain measurement is developed in this thesis. Furthermore, it is necessary to investigate the robustness of optimal sensor placement under parametric uncertainty considering different sensor location methodologies such as Fisher information matrix and based on energy matrix rank optimization.

Of course, the analysis of structural reliability and the quantitative evaluation of the quality of the hybrid experimental- numerical model used in complex structures utilization of the approaches for determining measurement uncertainties, which is experimental/monitoring data, which is developed in this thesis. Therefore, the uncertainties in the mechanical properties of materials and monitoring data, the methodology developed and presented in this thesis are rather sufficient.

Bibliography

[1] Oberkampf, L.W.; Trucanob, T.G.: Verification and validation in computational fluid dynamics. In: *Progress in Aerospace Sciences*, 38 (2002) 209-272.

[2] Chen, W.; Baghdasaryan, L.; Buranathiti, T.; Cao, J.: Model validation via uncertainty propagation and data transformation. In: *AIAA Journal*, Vol. 42, No. 7, July 2004.

[3] Dwoyer D.: The relation between computational fluid dynamics and experiment. In: *AIAA 17th Ground Testing Conference*, Nashville, TN, American Institute of Aeronautics and Astronautics, 1992.

[4] McGrattan, K.; Toman, B.: Quantifying the predictive uncertainty of complex numerical models. In: *Metrologia*, 48(3), 2011, 173-180.

[5] Keitel, H.; Dimmig-Osburg, A.; Vandewalle, L.; Schueremans, L.: Selecting creep models using Bayesian methods. In: *Materials and Structures*, 45(10), 2012, 1513-1533.

[6] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML: *Evaluation of measurement data - Guide to the expression of uncertainty in measurement. Joint Committee for Guides in Metrology*, JCGM 100:2008.

[7] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML: *Evaluation of measurement data - Supplement 1 to the Guide to the expression of uncertainty in measurement - Propagation of distributions using a Monte Carlo method. Joint Committee for Guides in Metrology*, JCGM 101:2008.

[8] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML: *Evaluation of measurement data - Supplement 2 to the Guide to the expression of uncertainty in measurement - Extension to any number of output quantities*, JCGM 102: 2011.

[9] Gläser, M.: Cycles of comparison measurements, uncertainties and efficiencies. In: *Measurement Science and Technology*, (2000), 11, 20-24.

[10] Hall, B.: Uncertainty calculations using object-oriented computer modeling. In: *Measurement Science and Technology*, 10(5) (1999), 380-386.

[11] Malcolm, S.; Loveday, T.G.; Johannes, A.: Tensile Testing of Metallic Materials. In: *A Review EU Project* (2004).

[12] Most, T.; Kanbe, T.: Reliability analysis of the bearing failure problem considering uncertain stochastic parameters. In: *Computers and Geotechnics* 37 (2010) 299-310.

[13] Keitel, K.; Dimmig-Osburgb, A.: Uncertainty and sensitivity analysis of creep models for uncorrelated and correlated input parameters. In: *Engineering Structures* 32 (2010) 3758-3767.

[14] Reuter, M.; Werner, F.: Evaluation of complex engineering models using model quality analysis. In: *Engineering Structures* 42 (2012), 410-419.

[15] Keitel, H.; Jung, B.; Motra, H.B.; Stutz, H.: Quality Assessment of Coupled Partial Models Considering Soil-Structure Coupling. In: *Engineering Structures* 59 (2014), 565-573.

[16] Scheiber, F.; Motra, H.B.; Legatiuk, D.; Werner, F.: Uncertainty-based Evaluation and Coupling of Mathematical and Physical Models. In: *Probabilistic Engineering Mechanics*, (2015) under Review.

[17] Vasquez, V.R.; Whiting, WE. B.: Accounting for Both Random Errors and Systematic Errors in Uncertainty Propagation Analysis of Computer Models Involving Experimental Measurements with Monte Carlo Methods. In: *Risk Analysis*, 25(6) (2005), 1669-1681.

[18] Motra, H.B.; Dimmig-Osburg, A.; Hildebrand, J.: Quality assessment of models with an application to cyclic creep prediction of concrete. In: *International Journal of Reliability and Safety*, Vol. 8, Nos. 2/3/4, (2014), 262-283.

[19] Schwer, L.E.: Guide for Verification and Validation. In: *Computational Solid Mechanics* V & V, 10 / ASME. (2006) Technical report 5, 6, 59.

[20] Stein, P.; Lahmer, T.; Bock, S.: Synthese und Analyse von gekoppelten Modellen im konstruktiven Ingenieurbau. In: *Bautechnik Sonderheft*, (2011), 8-11.

[21] Scheiber, F.; Motra, H.B.; Legatiuk, D.; Werner, F.: Model Coupling in Structural Engineering Application. In: *Proceedings of 12th International Probabilistic Workshop, IPW2014*, 4th - 5th November 2014, Weimar/Germany, ISBN: 978-3-95773-183-8, 283-292.

[22] Carpenter, J.E.(Ed.): Models of Concrete Structures. In: *ACI Committee 444, American Concrete Institute Annual Convention,* Dallas, TX (1972).

[23] Taguchi, G.: *The system of experimental design: engineering methods to optimize quality and minimize costs, vols. 1 & 2*, UNIPUB/Kraus International Publications, White Plains, NY, USA; 1987.

[24] Scheiber, F.; Motra, H.B.: Tragwerksmonitoring und numerische Simulation. In: *Bautechnik Sonderdruck "Modellqualitäten"* 90 (2013), 63-68.

[25] Scheiber, F.; Motra, H.B.: Ansatz zur hybriden Modellierung im Konstruktiven Ingenieurbau - Bewertete Kopplung von mathematischen und physikalischen Modell. In: *7th symposium Experimentelle Untersuchungen von Baukonstruktionen*, Technische Universität Dresden Germany, 5th September 2013, ISSN 1613-6934, 21-33.

[26] Keitel, K.: *Evaluation Methods for the Prediction Quality of Creep Models of Concrete*. Bauhaus-Universität Weimar, Dissertation (2011).

[27] Reuter, M.: *Multicriterial evaluation method for the prognosis quality of complex engineering models.* Bauhaus-Universität Weimar, Dissertation (2012).

[28] ISO IEC 17025:1999: *General Requirements for the Competence of Calibration and Testing Laboratories.* (1999) International Organization for Standardization, Geneva.

[29] Weise, K.; Woeger, W.: A Bayesian theory of measurement uncertainty. In: *Measurement Science and Technology*, 4 (1) (1993), 1-11.

[30] Elster, C.; Wöger, W.; Cox, M.G.: Draft GUM Supplement 1 and Bayesian analysis. In: *Metrologia*, 44(2007), L31-2.

[31] Wübbeler, G.; Krystek, M.; Elster, C.: Evaluation of measurement uncertainty and its numerical calculation by a Monte Carlo method. In: *Measurement Science and Technology*, 19 (2008), 1-5.

[32] Elster, C.; Toman, B.: Bayesian uncertainty analysis under prior ignorance of the measurand versus analysis using the supplement 1 to the guide: a comparison. In: *Metrologia* 46 (2009) 261-266.

[33] Kacker, R.N.; Jones, A.: On the use of Bayesian statistics to make the guide to the expression of uncertainty in measurement consistent. In: *Metrologia* 40 (2003), 235-248.

[34] Lira, I.; Grientschnig, D.: Equivalence of alternative Bayesian procedures for evaluating measurement uncertainty. In: *Metrologia* 47 (2010), 334-336.

[35] Possolo, A.S.; Toman, B.: Assessment of measurement uncertainty via observation equations. In: *Metrologia* 44 (2007), 464-475.

[36] Motra, H.B.; Dimmig-Osburg, A.; Hildebrand, J.: Quality assessment of strain measurement in concrete structures. In: *Bautechnik-Special Edition, Model quality*, 90 (2013), 69-75.

[37] Lira, I. : *Evaluating the Uncertainty of Measurement: Fundamentals and Practical Guidance* (2002), Bristol: Institute of Physics Publishing.

[38] Jaynes, E.T.: *Probability Theory: The Logic of Science* (2003), Cambridge: Cambridge University Press.

[39] Gelman, A.; Carlin, J.B.; Stern, H.S.; Rubin, D.B.: *Bayesian Data Analysis* 2nd edn. (2003), Boca Raton, FL: Chapman and Hall/CRC.

[40] Saltelli, A.; Ratto, M.; Andres, T.; Campolongo, F.; Cariboni, J.; Gatelli, D.; Saisana, M.; Tarantola, S.: *Global Sensitivity Analysis. The Primer.* John Wiley and Sons, 2008 2, 7, 8, 131. 132

[41] Meerovich Sobol, I.: Sensitivity estimates for nonlinear mathematical models. In: *Mathematical Modeling and Computational Experiments*, 1(4), 1993, 407-414.

[42] Janssen, P.H.M.: Assessing sensitivities and uncertainties in models: a critical evaluation. J. Grasman and G. van Straten, editors, In: *Predictability and Nonlinear Modelling in Natural Science and Economics*, pages 344-361. Kluwer Academic Publishers, 1994.

[43] Kleijnen, P.C.J.: Sensitivity analysis versus uncertainty analysis: When to use what. In J. Grasman and G. van Straten, editors, In: *Predictability and Nonlinear Modelling in Natural Science and Economics*, pages 322-331. Kluwer Academic Publishers, 1994.

[44] Baumgarter, T.A.: Norm-referenced measurement: reliability in: Safrit, M J and Wood, T M (eds), In: *Measurement Concepts in Physical Education and Exercise Science*, Champaign, Illinois (1989), 45-72.

[45] Farrell, T.; Cairns, M.; Leslie, J.: Reliability and validity of two methods of three-dimensional cervical volume measurement. In: *Ultrasound Obstet Gynecol* (2003); 22, 49-52.

[46] Bland, J.M.; Altman, D.G.: Measuring agreement in method comparison studies. In: *Stat Methods Med Res* (1999) 8, 135-160.

[47] Bland, J.M.; Altman, D.G.: Statistical methods for assessing agreement between two methods of clinical measurement. In: *Lancet* (1986) 1, 307-310.

[48] Motra, H.B.; Hildebrand, J; Wuttke, F.: Monte Carlo method for the evaluation of measurement uncertainty: application to determine material properties. *Probabilistic Engineering Mechanics*, (2015) under Review.

[49] Schmucker, D.G.: Models, Models, Models: The Use of Physical Models to Enhance the Structural Engineering Experience. In: *Proceedings of the 1998 American Society for Engineering Education Annual Conference and Exposition*, Seattle, WA, June 28-July 1, 1998.

[50] Kresta, S.M.: Hands-on Demonstrations: An Alternative to Full Scale Lab Experiments. In: *Journal of Engineering Education*, Vol. 87, No. 1, 1998, 7-9.

[51] Harris, H.G.; Sabnis, G.M.: *Structural Modeling and Experimental Techniques.* CRC Press, Washington, D.C.(1999).

[52] Janney, J.R.; Breen, J.E.; Geymayer, H.: Use of models in structural engineering. In: *Models for concrete structures*, ACI SP-24, American Concrete Institute, Detroit, MI, 1970, 1-18.

[53] P103d - Annex: *Policy on Estimating Measurement Uncertainty for Construction Materials & Geotechnical Testing Labs.* The American Association for Laboratory Accreditation (2010).

[54] Meyer, P.L.: *Introductory Probability and Statistical Applications* 2nd edn (1970),Reading, MA: Addison-Wesley.

[55] Pavese, F.; Forbes, A.B.: *Data Modeling for Metrology and Testing in Measurement Science.* Springer publication, 2008.

[56] Lira, L.; Wöger, W.: Comparison Between the Conventional and Bayesian Approaches to Evaluate Data. In: *Metrologia*, 43, (2006), 249-259.

[57] EA-4/16: *Guidelines on the expression of uncertainty in quantitative testing.* European cooperation of accreditation, 2004 French and English version available on the LNE web site www.lne.fr

[58] Walter, E.; Pronzato, L.: *Identification of Parametric Models from Experimental Data.* Springer Verlag, Heidelberg, Germany, 1999.

[59] Bonfiglioli, B.; Di Muro P.; Pascale, G.: Reliability of strain measurements in FRP. In: *Proceedings of the Structural Health Monitoring*, SHM-ISIS2002 Workshop, 19-20 September 2002, 275-83.

[60] Kim, S.; Frangopol, D.M.: Optimal planning of structural performance monitoring based on reliability assessment. In: *Probabilistic Engineering Mechanics*, 25(1),2010, 86-98.

[61] Strauss, A.; Frangopol, D:M:; Kim, S.: Use of monitoring extreme data for the performance prediction of structures: Bayesian updating. In: *Engineering Structures,* 30(12), 2008, 3654-3666.

[62] William, L.O.; Matthem, F.B.: Measures of agreement between computation and experiment: Validation metrics. In: *Journal of Computational Physics* 217 (2006), 5-36.

[63] Babusak, I.; Oden, J.T.: Verification and Validation in Computational Engineering and Science: Basic Concepts. In: *Computational Methods in Applied Mechanics and Engineering* 193 (2004), 4057-4066

[64] Cronbach, L.J.; Gleser, G.C.; Nanda, H.; Rajaratnam, N.: *The Dependability of Behavioral Measurements*. Wiley, New York (1972).

[65] Pendrill, L.R: Using measurement uncertainty in decision-making and conformity assessment. In: *Metrologia* 51 (2014), 206-218.

[66] International Vocabulary of Basic and General Terms in Metrology, 2nd ed., ISO (1993).

[67] Bich, W.; Cox, M.G.; Harris, P.M.: Evolution of the "Guide to the Expression of Uncertaity in Measurement". In: *Metrologia*, 43 (4), 2006, 161-166.

[68] Dietrich, C.F.: *Uncertainty, Calibration and Probability- the statistics of scientific and industrial measurement*, Published by Adam Hilger, London, 1973.

[69] Wojtkiewicz, S.F.; Eldred, M.S.; Field, R.V.; Urbina, A.; Red-Horse, J.R.: Uncertainty quantification in large computational engineering models. In: *AIAA,* 2001.

[70] Pellissetti, M.F.; Pradlwarter, H.J.; Schueller, G.I.: Relative importance of uncertain parameters in aerospace applications. In: *III European Conference on Computational Mechanics Solids, Structures and Coupled Problems in Engineering,* Lisboa, Portugual, 2006.

[71] International Organization for Standardization, *Guide to the Expression of Uncertainty in Measurement*, International Organization for Standardization, Geneva, Switzerland

[72] Taylor, B.N.; Kuyatt, C.E.: *Guidelines for Evaluation and Expressing Uncertainty of NIST Measurement Results, Technical Note 1297*, United States National Institute of Standards and Technology (NIST), United States Department of Commerce, Washington, DC (1994).

[73] Adams, T.M.: *G104 - A2LA Guide for the Estimation of Measurement Uncertainty in Testing*, 2002.

[74] Lord, G.J.; Wright, L.: Uncertainty Evaluation in Continuous Modeling. *NPL Report CMSC 31/03*, 2003.

[75] Krystek, M.: *Berechnung der Messunsicherheit: Grundlagen und Anleitung für die praktische Anwendung*. Beuth Verlag, 2012.

[76] Sommer, K.D.; Siebert, B.R.L.: Systematic approach to the modelling of measurements for uncertainty evaluation. In: *Metrologia* 43:(2006), 200-210.

[77] Cox, M.G; Harris, P.M.: *Software Support for Metrology Best Practice Guide No 6, Uncertainty Evaluation.* NPL Report DEM-ES-011. National Physical Laboratory, Teddington, UK, Sept 2006.

[78] Kacker, R.; Jones, A.: On use of Bayesian statistics to make the Guide to the Expression of Uncertainty in Measurement consistent. In: *Metrologia* 40 (2003), 235-48.

[79] Box, G.E.P.; Tiao, G.C.: *Bayesian Inference in Statistical Analysis.* Reading, MA: Addison-Wesley,Reprinted 1992 Wiley Classics Library Edition, 1973.

[80] Bretthorst, G.L.: *Bayesian Spectrum Analysis and Parameter Estimation.* Berlin: Springe, 1988.

[81] Kacker, R.N.: Bayesian alternative to the ISO-GUM's use of the Welch-Satterthwaite formula. In: *Metrologia* (2006) 43, 1-11.

[82] Cox, M.G.; Siebert, B.R.L.: The use of a Monte Carlo method for evaluating uncertainty and expanded uncertainty. In: *Metrologia* 43(2006), 178-88.

[83] Cox, M.G.; Harris, P.M.: *Software specifications for uncertainty evaluation.* Technical Report DEM-ES-010, National Physical Laboratory, Teddington, UK, 2006.

[84] HBM: *Dokumentation Messverstärkersystem.* HBM MGCplus (2008).

[85] Thöns, S. *Monitoring based condition assessment of offshore wind turbune support structures.* PhD Dissertation ETH Zurich (2011) .

[86] Vishay: Micro-Measurements, Technical note TN-504-1, *Strain gauge thermal output and gauge factor variation with temperature.* 2007, 35-47.

[87] Allard, A.; Fischer, N.: Recommendation tools for sensitivity analysis associated to the evaluation of measurement uncertainty. In: *Advanced Mathematical and Computational Tools in Metrology IX,* (2009), 1-12.

[88] Mario, S.; Hogue, I.B.; Ray, C.J.; Kirscher, D.E.: A Methodology For Performing Global Uncertainty And Sensitivity Analysis In Systems Biology. In: *J Theor Biol.*, 2008, 454(1), 178-196.

[89] Kessel, R.; Kacker, R.; Berglund, M.: Coefficient of contribution to the combined standard uncertainty. In: *Metrologia*, 43(4), 2006, 189-195.

[90] Motra, H.B.; Dimmig-Osburg, A.; Hildebrand, J.: Evaluation of Experimental Measurement Uncertainty in Engineering Properties of PCC Samples. In: *Journal of Civil Engineering Research*, Vol. 3 No.3, 2013, 104-113.

[91] Motra, H.B.; Hildebrand, J.; Dimmig-Osburg, A; Lahmer, T.: Reliability of Measurement Results for Materials Testing. In: *Proceedings of 12^{th} International Probabilistic Workshop, IPW2014*, 4^{th} - 5^{th} November 2014, Weimar/Germany, ISBN: 978-3-95773-183-8, 228-237.

[92] Tang, Y.; Reed, P.; van Werkhoven, K.; Wagener, T.: Advancing the identification and evaluation of distributed rainfall-runoff models using global sensitivity analysis. In: *Water Resour. Res.* 43, W06415, 2007, 1-14.

[93] McKay, M.D.; Beckman, R.J.; Conover, W.J.: A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. In: *Technometrics* 42 (1) 2000, 55-61.

[94] Schneider, J.: *Sicherheit und Zuverlässigkeit im Bauwesen*, B.G. Teubner Verlag, Stuttgart (1994).

[95] Fischer, L.: Sicherheitskonzept für neue Normen, Teil 9: Einführung in die Zuverlässigkeitstheorie tragender Baukonstruktionen. In: *Bautechnik* 76 (1999), Heft 10, 921-931.

[96] Antiny, J.: *Design of Experiments for Engineers and Scientists.* Butterworth-Heinemann, Elsevier, Publication, 2003.

[97] Robert, L.M., Richard, F.G.T: *Statistical Design and Analysis of Experiments With Applications to Engineering and Science.* A John Wiley and Sons Publication, 2003.

[98] Sing, J.: *Comparative Analysis of Robust Design Method.* PhD Dissertation, MIT, 2006.

[99] DIN EN ISO/IEC 17025, Ausgabe 04.005: *General requirements for the competence of testing and calibration laboratories.* Beuth Verlag, Berlin, August 2005.

[100] ISO/IEC Guide 43-1:1997: *Proficiency Testing by Interlaboratory Comparisons. Part 1: Development and operation of proficiency testing schemes.* International Organisation for Standardization, Genève, First Edition 1997.

[101] Pauels, J.; Lamnrty, A.; Schimmel, H.: Homogeinity testing of reference materials. In: *Accreditation and Quality Assurance* 3 (1998), 51-55.

[102] DIN ISO 13528:2009: *Statistische Verfahren für Eignungsprüfungen durch Ringversuche (ISO 13528:2005).* Beuth Verlag, Berlin, Januar 2009.

[103] DIN EN ISO/IEC 17043:2010: *Konformitätsbewertung - Allgemeine Anforderungen an Eignungsprüfungen.* Beuth Verlag, Berlin, Mai 2010.

[104] Report No. 558, *Tensile Testing of Metals Proficiency Testing Program.* NSW 2128 Australia, 2007.

[105] Motra, H.B.; Hildebrand, J.; Dimmig-Osburg, A.; Stutz, H.: A probabilistic method for quality evaluation of experiments in Civil Engineering. In: *Measurement*, Journal of the International Measurement Confederation (IMEKO), under Review.

[106] Strauss, A.; Corbin, J.: *Basics of Qualitative Research: Grounded Theory Procedures and Techniques.* Newbury Park, CA, SAGE Publications, 1990.

[107] Gall, M.D.; Borg, W.R.: *Educational Research: An Introduction.* White Plains, NY, Longman Publishers, 1996.

[108] Lee, K.H.; Park, G.; Lee, W.: Robust Design for unconstrained optimization problems using the Taguchi Method. In: *AIAA Journal*, Vol. 34, No. 5, 1996.

[109] Du, X.; Chen, W.: Methodology for managing the effect of uncertainty in simulation-based design.In: *AIAA Journal*, Vol. 38, No. 8, 2000.

[110] Lira, I.; Kyriazis, G.: Bayesian inference from measurement information. In: *Metrologia* 36 (1999), 163-9.

[111] ISO 6892-1:2009E: *International Standard, Metallic materials-Tensile testing-Part 1:* technique of test at room temperature, 2009.

[112] Lord, J.D.; Morrell, R.M.: Elastic modulus measurement-obtaining reliable data from the tensile test. In: *Metrologia* 47 (2010), 502-515.

[113] Geomagic *Geomagic Studio 9 for 3-D Shape Measurement.* Quick stard guid, 2006.

[114] Zhang, S.: *High-resolution, Real-time 3-D Shape Measurement.* PhD dissertation, Stony Brook University, 2005.

[115] Motra, H.B.; Hildebrand, J.; Dimmig-Osburg, A.: Assessment of Strain Measurement Techniques to Characterize Mechanical Properties of Structural Steel. In: *Engineering Science and Technology: an International Journal*, Vol. 17 (2014) 260-269.

[116] Morta, H.B.; Hildebrand, J.; Dimmig-Osburg, A.: Influence of specimen dimensions and orientations on the tensile properties of structural steel. In: *Material Testing*, Vol. 56, No. 11-12, (2014) 929-936.

[117] Paul, S.P.: X-Ray diffraction residual stress technique. In: *Metals Handbook. 10. Metals Park: American Society for Metals*,1986, 380-392.

[118] Hand, N.N.: Handbook of Measurement of Residual Stresses. Society for Experimental Mechanics. In: *Society for Experimental Mechanics Inc.*, 1996.

[119] Bell, S.: *A Beginner's Guide to Uncertainty NPL Measurement Good Practice Guide no. 11.* 1999.

[120] Birch, K.: *Estimating Uncertainties in Testing NPL Measurement Good Practice Guide no. 36.* 2001.

[121] Kandil, F.A. et al (ed): *UNCERT Manual. Manual of codes of practice for the determination of uncertainties in mechanical tests National Physical Laboratory.* 2000.

[122] Dyson, B.F.; Loveday, M.S.; Gee M.G. (ed): *Aspects of modulus measurements Aspects of Materials Metrology and Standards for Structural Performance.* (1994) (London: Elsevier Applied Science) chapter 8.

[123] Lord, J.D.; Loveday, M.S.; Rides, M.; McEnteggart, I: *TENSTAND WP2 Report - Digital Tensile Software Evaluation.* July 2004.

[124] Klingelhöffer, H.; Ledworuski, S.; Brookes, S.; May, T.: *TENSTAND WP4 Report - Validation of the Machine Control Characteristics.* Sept 2004.

[125] Bahn, G.W.; Kim, J.J.; Lee, H.M.; Huh, Y.H.: Establishment of traceability in the measurement of the mechanical properties of materials. In: *Metrologia*, 47, 2010, 32-40.

[126] Klingelhöffer, H.; Ledworuski, S.; Brookes, S.P.; May, T.: Computer controlled tensile testing according to EN 10002-1, Results of a comparison test programm to validate a proposal for an amendment of the standard. In: *Final report of the European project TENSTAND - Work Package 4*, 2005.

[127] Flohr, A.: *Stoffliche Aspekte des Einflusses einer Polymermodifikation auf die statischen und dynamischen Eigenschaften von Konstruktionsbeton.* Diploma's thesis, Bauaus- Universität Weimar, 2005.

[128] DIN 1048-5, 1981: *Prüfverfahren für Beton - Festbeton, gesondert hergestellte Probekörper.*

[129] ACI 214R-02: Evaluation of Strength Test Results of Concrete. : In *Reported by ACI Committee 214*, 2002.

[130] ASTM C39: *Compressive Strength of Cylindrical Concrete Specimens.* 2005.

[131] Sac-singnals technical guide 3: *Guidance on Measurement Uncertainty for Civil Engineering and Mechanical Testing Laboratories.* 2007.

[132] NATA: *Guide to Measurement Uncertainty for Construction Materials Testing.* 2011.

[133] Flack, D.: *Calipers and Micrometers, In Measurement Good Practice Guide No. 40. Teddington.* (2001), Middlesex, UK: National Physical Laboratory.

[134] Dumanić, E.; Sekulić, D.: Measurement uncertainty of test method for concrete cubes tensile strength. In: *Proceedings of the 18th Metrology Symposium,* 8-10 October 2001. Cavtat, Croatia, 138-141.

[135] Gabauer, W.: *UNCERT Code of Practice 07, The Determination of Uncertainties in Tensile Testing.* Linz, Austria, 2000.

[136] Lacasse, S.; Nadim, F.: Uncertainties in characteristic soil properties. In: *Uncertainty in the Geological Environment*, ASCE specialty conference, Madison, WI. ASCE, Reston, VA, 1996:40-75

[137] Phoon, K.K.; Kulhawy, F.H.: Evaluation of geotechnical property variability. In: *Canadian Geotechnical Journal*, 1999(36), 625-639.

[138] Huber, M.: *Soil variability and its consequences in geotechnical engineering.* Institut für Geotechnik der Universität Stuttgart, Dissertation (2013).

[139] Knabe, T.; Datcheva, M.; Lahmer, T.; Cotecchia, F.; Schanz, T.: Identification of constitutive parameters of soil using an optimization strategy and statistical analysis. In: *Computers and Geotechnics* 49 (2013) 143-157.

[140] DIN 18137-2, 2011-04: *Baugrund, Versuche und Versuchsgeräte - Bestimmung der Scherfestigkeit - Teil 2: Triaxialtest.* 2011.

[141] DIN 18137-2:1990-2 Baugrund, Untersuchung von Bodenproben - Bestimmung der Scherfestigkeit - Teil 2: *Triaxialversuch*, 1990.

[142] Frangopol, D.M; Strauss, A.; Kim, S.: Use of monitoring extreme data for the performance prediction of structures: General approach. In: *Engineering Structures* 30 (2008) No. 12:3644-3653.

[143] Frangopol, D.M.; Strauss, A.; Kim, S.: Bridge reliability assessment based on monitoring. In: *J. Bridge Engineering*, ASCE 13 (2008) No. 3, 258-270.

[144] Thones, S.; Faber, M.H.; Rücker, W.; Rohrmann, R.G.: Assessment and monitoring of reliability and robustness of offshore wind energy converters. In: *ESREL 2008 and 17th SRA-Europe Conference.* Valencia, Spain, 1567-1575.

[145] Bonfiglioli, B.; Pascale, G.; Martine de Mingo, S.: Dynamic testing of reinforced concrete beams damaged and repaired with CFRP sheet. In: *Journal of Materials in Civil Engineering*, ASCE, 16, (2004) No. 5, 400-406.

[146] Zimmermann, T.; Haider, K.; Strauss, A.: Measurement redundancy incorporating measurement uncertainty. In: *Proceedings of the 11th International Probabilistic Workshop,* Brno 2013.

[147] Chalmers, G.F.: The dangers of the 5% mentality. In: *Proceedings of the International Conference of the BSSM and The Institution of Production Engineers*, 1980.

[148] Window, A.L.; Holister, G.S.: Strain gauge technology, 2nd ed. In: *Applied Science Publishers*, London, UK 1992.

[149] Veen, A.M.V.; Cox, M.G.: Error analysis in the evaluation of measurement uncertainty. In: *Metrologia* (2003) 40, 42-50.

[150] Motra, H.B.; Dimmig-Osburg, A.; Hildebrand, J.: Influence of Measurement Uncertainties on Results of Creep Prediction of Concrete under Cyclic Loading. In: *Proceedings of the 8th International Conference on Fracture Mechanics of Concrete and Concrete Structures*, March 10-14, 2013 /Toledo - Spain, ISBN 978-84-941004-1-3, 805-814.

[151] Motra, H.B.; Dimmig-Osburg, A.; Hildebrand, J.: Probabilistic assessment of concrete creep models under repeated loading considering measurement uncertainty. In: *10th International Probabilistic Workshop*, Stuttgart, ISBN 978-3-921837-67-2, 2012, 285-301.

[152] TML, TML WFLA-6-17: *Strain gauge test data* (2008).

[153] Keil, S.: *Beanspruchungsermittlung mit Dehnungsmessstreifen.* (1995) Cuneus.

[154] JCSS: *Probabilistic Model Code*, JCSS Joint Committee on Structural Safety (2006).

[155] VDI/VDE/GESA. *Experimental structure analysis, Metallic bonded resistance strain gages, Characteristics and test conditions.* VDI/VDE/GESA 2635, (2007) Part 1.

[156] Bäumel, A.: Messen mit DMS - Grundlagen (Measuring with electrical strain gages). In: *Proc., Hottinger Baldwin Measuring Technique, Seminar - Unterlagen Hottinger Baldwin Messtechnik*, Vienna, Austria, 2006.

[157] Vishay: Errors due to misalignment of strain gages. In: *Technical Note No. TN-511*. http://www.vishay.com/docs 11519/ weldpat.pdf, 2007.

[158] Hesse, T.: Messunsicherheiten - auch der Messverst rkeräträgt dazu bei (Uncertainties in measuring Also the amplifier participates). In: *Proc., Hottinger Baldwin Measuring Technique, Seminar-Unterlagen Hottinger Baldwin Messtechnik*, Vienna, Austria, 2006.

[159] DMS-Katalog: *Hottinger Baldwin Messtechnik (Hottinger Baldwin measuring technique)*. http://www.hbm.com, 2007.

Appendix

Appendix A: Derivatives of complex multivariate measurement functions

A.1 In this annex consideration is given to the algebraically efficient determination of the partial derivatives of first order of the measurement function f in a complex multivariate measurement model

$$Y = f(X),$$

where,

$$X = (X_1, ..., X_N)^T, \qquad Y = (Y_1, ..., Y_m)^T),$$

and

$$f = (f_1, ..., f_m)^T),$$

X_i denoting the complex quantity $X_{i,R} + iX_{i,I}$, with $X_{i,R}$ and $X_{i,I}$ real scalar quantities, and $i^2 = -1$, and similarly for Y_j and f_j.

A.2 Let U_x denote the covariance matrix of dimension $2N\ X\ 2N$ associated with an estimate x of X. U_x takes the from

$$U_x = \begin{bmatrix} \mu(x_1, x_1) & \cdots & \mu(x_1, x_N) \\ \vdots & \ddots & \vdots \\ \mu(x_N, x_1) & \cdots & \mu(x_N, x_N) \end{bmatrix}$$

where

$$U_{x_i,x_j} = \begin{bmatrix} \mu(x_{i,R}, x_{j,R}) & \mu(x_{i,R}, x_{j,I}) \\ \mu(x_{i,I}, x_{j,R}) & \mu(x_{i,I}, x_{j,I}) \end{bmatrix}$$

is the covaraince matrix of dimension $2\ X\ 2$ associated with the estimates x_i and x_j of X_i and X_j, respectively.

A.3 The covariance matrix

$$U_y = \begin{bmatrix} \mu(y_1, y_1) & \cdots & \mu(y_1, y_m) \\ \vdots & \ddots & \vdots \\ \mu(y_m, x_1) & \cdots & \mu(y_m, x_m) \end{bmatrix}$$

of dimension $2m\ X\ 2m$, where

$$U_{y_l,x_j} = \begin{bmatrix} \mu(y_{l,R}, y_{j,R}) & \mu(y_{l,R}, y_{j,I}) \\ \mu(y_{l,I}, y_{j,R}) & \mu(y_{l,I}, y_{j,I}) \end{bmatrix}$$

associated with the estimate

$$y = f(x)$$

of Y is given by the generalized law of propagation of uncertainty

$$U_y = c_x U_x c_x^T$$

A.4 c_x is the sensitivity matrix of dimension $2m\ X\ 2N$ given by evaluating

$$c_x = \begin{bmatrix} C_{1,1} & \cdots & C_{1,N} \\ \vdots & \ddots & \vdots \\ C_{m,1} & \cdots & C_{m,N} \end{bmatrix}$$

at $X = x$, where $C_{j.i}$ is the matrix of dimension $2\ X\ 2$ of the partial derivatives of first order of the real and imaginary parts of f_j with respect to the real and imaginary parts of X_i:

$$C_{j,i} = \begin{bmatrix} \frac{\partial f_{j,R}}{\partial X_{i,R}} & \frac{\partial f_{j,R}}{\partial X_{i,I}} \\ \frac{\partial f_{j,I}}{\partial X_{i,R}} & \frac{\partial f_{j,I}}{\partial X_{i,I}} \end{bmatrix}$$

A.5 For any complex scalar quality $Q = Q_R + iQ_I$, consider the matrix representation of dimension 2 x 2 for Q:

$$M_(Q) = \begin{bmatrix} Q_R & -Q_I \\ Q_I & Q_R \end{bmatrix}$$

Then, $C_{j,i}$ can be exprssedx as

$$C_{j,i} = M\left(\frac{\partial f_j}{\partial X_i}\right),$$

and provides the basis for an algebraically fficient means for forming the partial derivatives: only the complex derivatives of first order of the f_j with respect to the X_i need be formed.

Appendix B: Testing and Evaluation Results

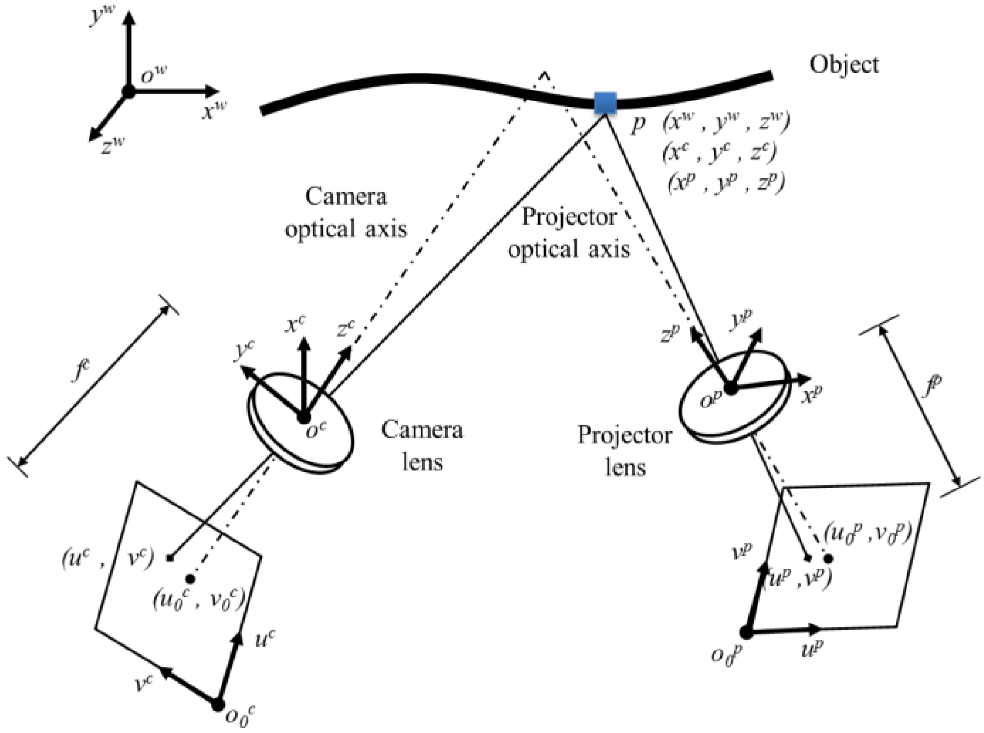


Figure Annex-B.1: Structured light 3D scanner configuration [114]

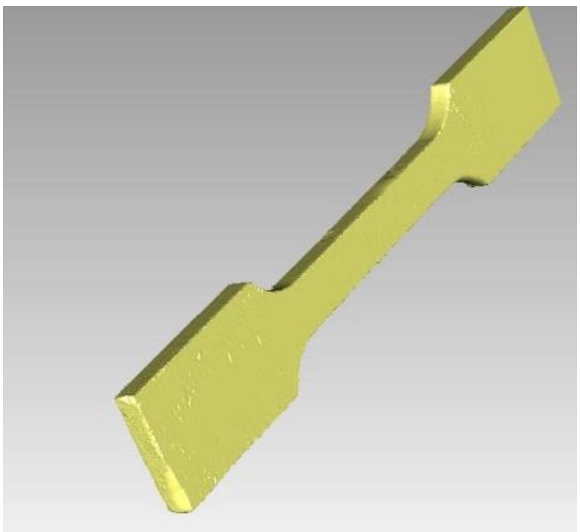


Figure Annex-B.2: Tensile specimen scanning

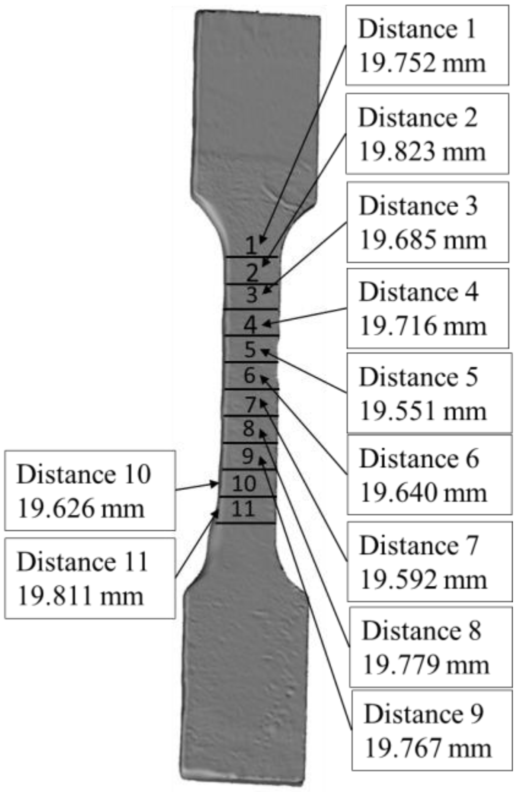


Figure Annex-B.3: Projected point-distance definition

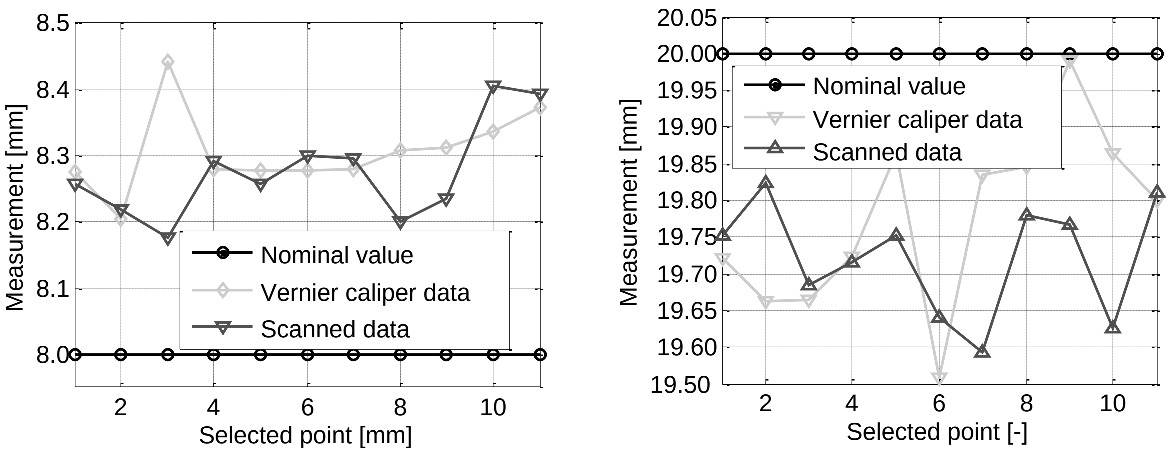


Figure Annex-B.4: Results of the 20 mm and 8 mm thickness block scanning 11 different points in same sample

Table Annex-B.1. Statistical calculation of 3d scanned and vernier caliper data

	Mean		Std.dev		Variance	
	Scanned	Vernier	Scanned	Vernier	Scanned	Vernier
Width 20 mm	20.00	20.13	0.34	0.39	0.11	0.15
Thickness 8 mm	8.60	8.633	0.14	0.11	0.02	0.01
Width 40 mm	40.20	40.35	0.22	0.26	0.05	0.07
Thickness 8.6 mm	8.79	8.85	0.31	0.30	0.10	0.09

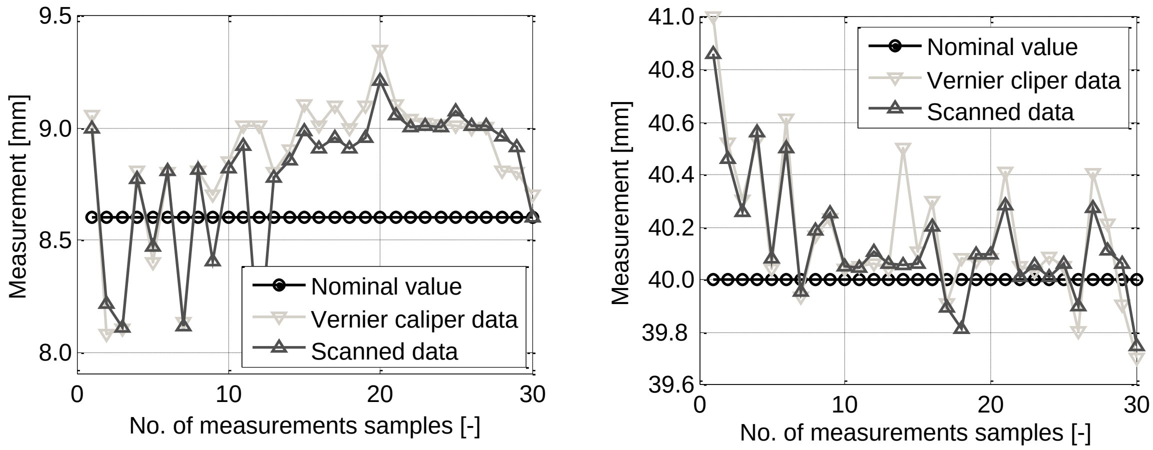


Figure Annex-B.5: Results of the 20 mm and 8 mm thickness block scanning 11 different points in same sample

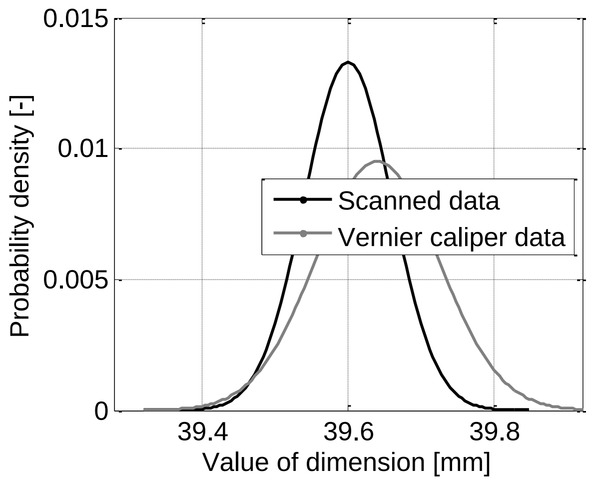


Figure Annex-B.6: Probability densities of the measurement uncertainty mean of 3D scanned and vernier caliper data.

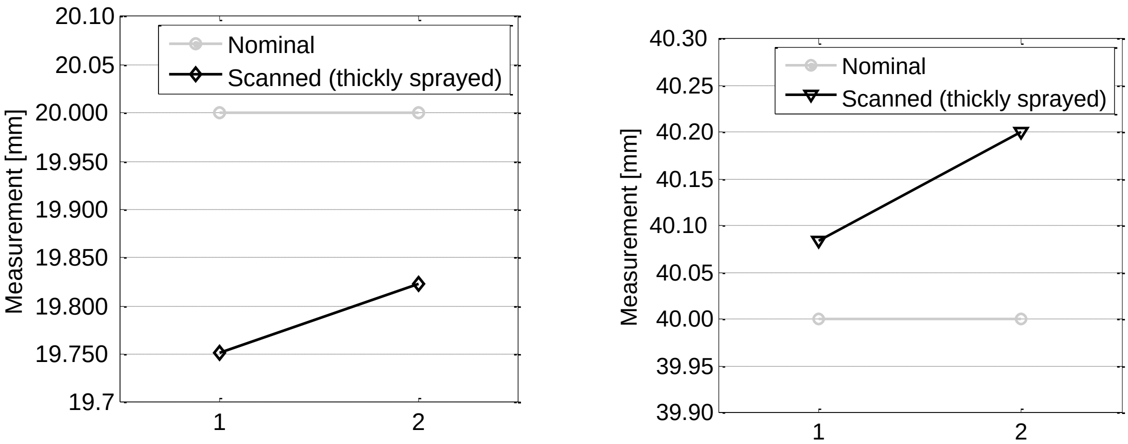
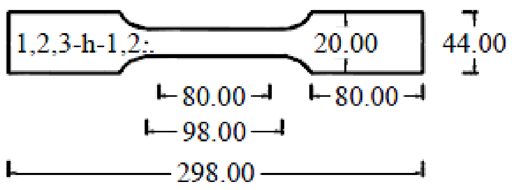
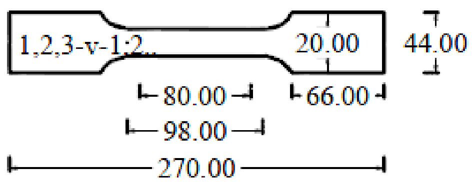


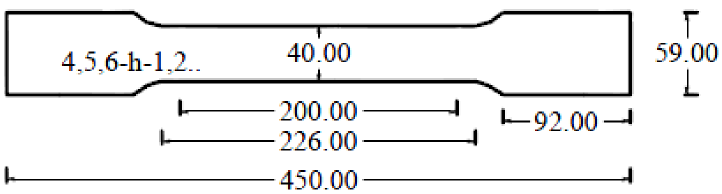
Figure Annex-B.7:Results of thickly spayed block scanning 20 mm width, 40 mm width



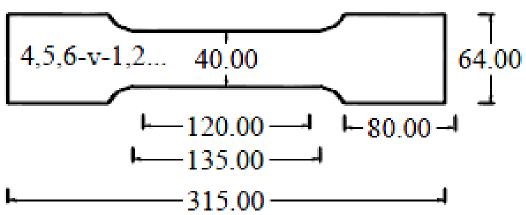
Specimen dimension for type 1 (ID1) test piece of IPE 360, horizontal direction



Specimen dimension for type 2 (ID2) test piece of IPE 360, vertical direction



Specimen dimension for type 3 (ID3) test piece of IPE 400, horizontal direction



Specimen dimension for type 4 (ID4) test piece of IPE 400, vertical direction

Figure Annex-B.8: Dimension of specimens [mm]

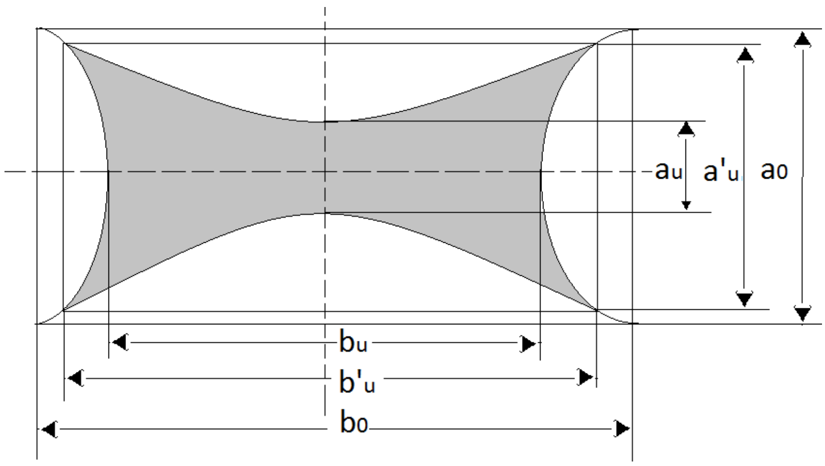


Figure Annex-B.9: Dimensions of the rectangular specimen at fracture point; a_0 - thickness of the specimen before fracture; b_0 - width of the specimen before fracture; a_u and a'_u - the minimum and maximum thickness of the specimen at fracture point respectively; b_u and b'_u - the minimum and maximum width of the specimen at fracture point respectively

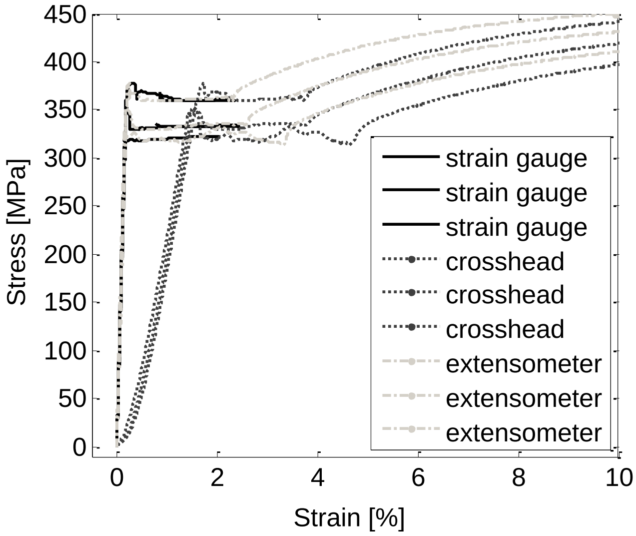


Figure Annex-B.10: A systematic diagram of the stress strain curve, showing the expected Young' modulus value and values calculated and recorded from the tensile tests conducted

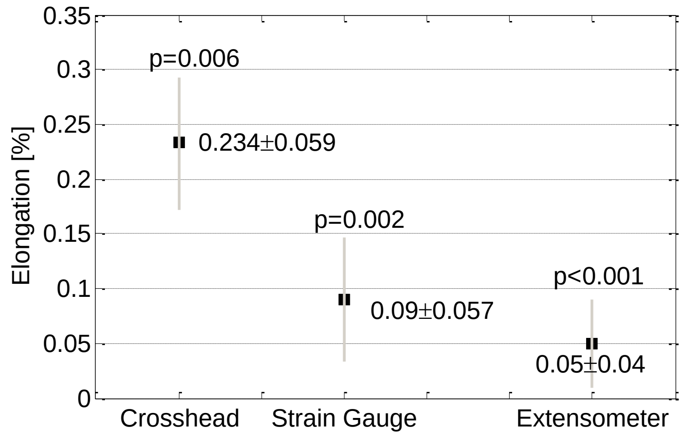


Figure Annex-B.11: A Elongation results for three different strain measurement techniques. The strain calculated from crosshead strain were significantly greater than the strain gauge and extensometer (p<0.001, p=0.002, and p=0.006 respectively, n = 20)

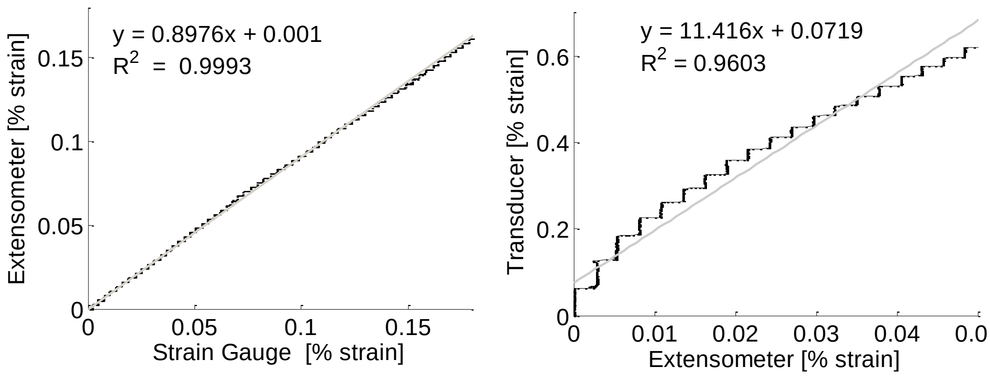


Figure Annex-B.12: Expressing regressing uncertainty of different stain measurement methods, showing the expected Young' modulus value and values calculated and recorded from the tensile tests conducted

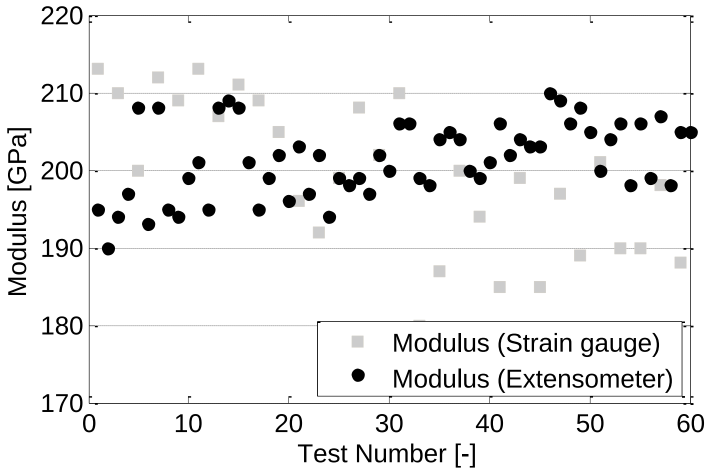


Figure Annex-B.13: Variation in modulus, measured in lab and different methods of strain measurement

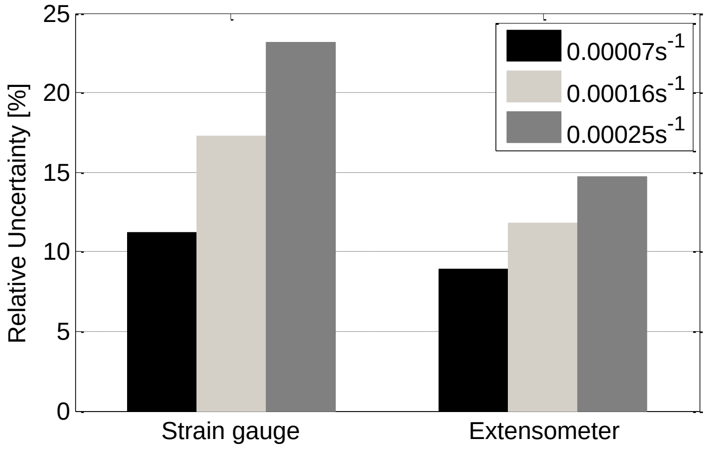


Figure Annex-B.14: Uncertainty in modulus: two strain measurement methods and three test conditions for group 1 specimens

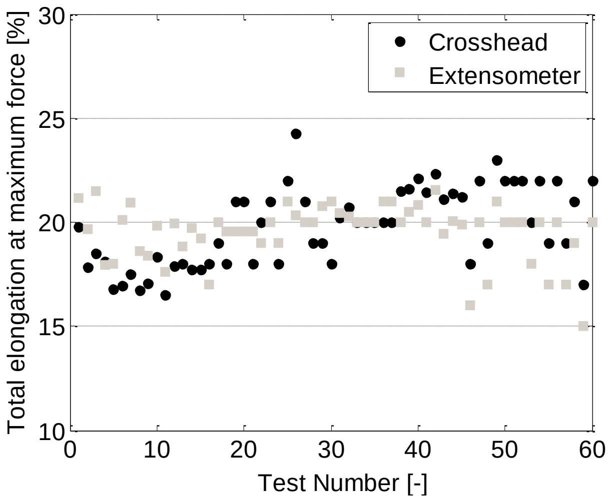


Figure Annex-B.15: Percentage elongation at maximum force:two strain measurement techniques

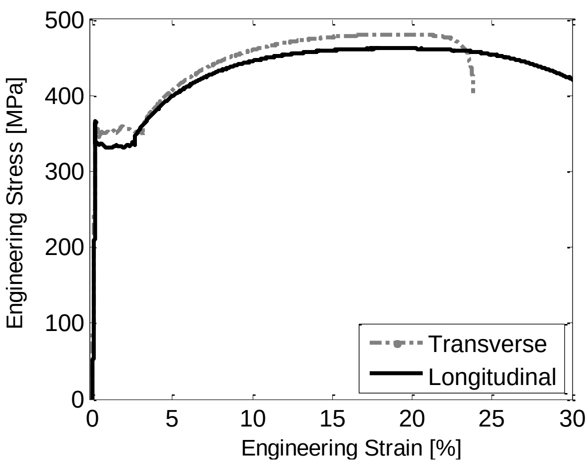


Figure Annex-B.16: Engineering stress (MPa) versus engineering strain (%) curve for ID1 and ID2 and strain rate $0.00016s^{-1}$

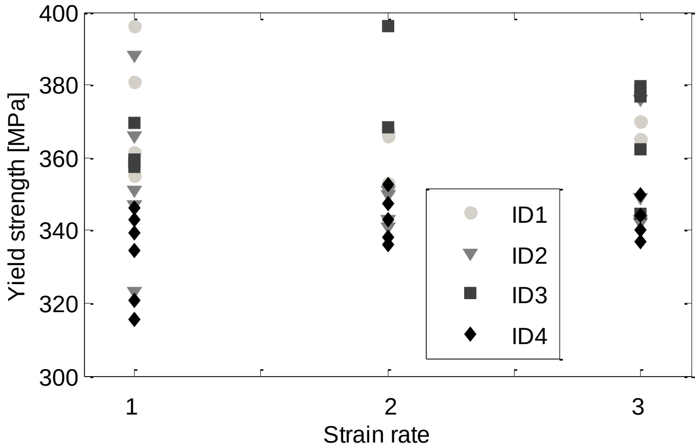


Figure Annex-B.17: Specimen size and strain rate effect in strength for strain rate (1, 2, 3)

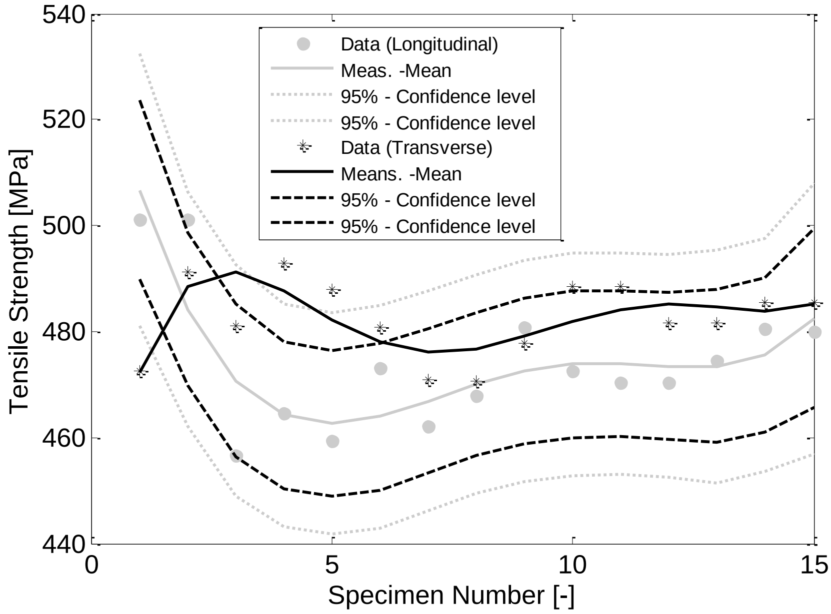


Figure Annex-B.18: Effect of orientation on tensile strength

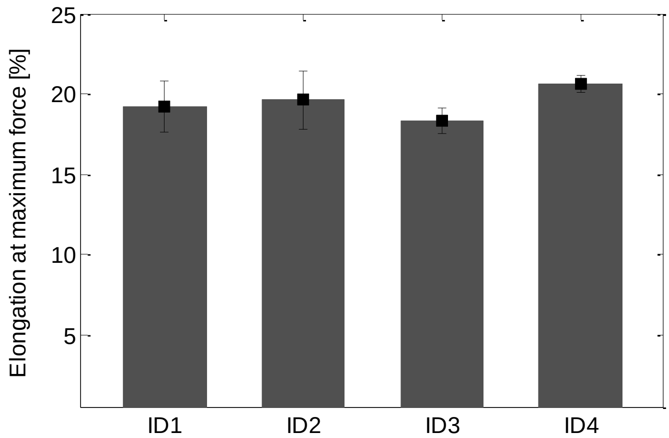


Figure Annex-B.19: Effect of geometry and orientation on elongation at maximum force

Table Annex-B.2: Sample of data analysis

Sample No.	Speed mm/s	$R_{p0.2}$ [MPa] machine	$R_{p0.2}$ [MPa] Strain Gauge	$R_{p0.2}$ [MPa] Extensometer	R_{eH}[MPa] machine	R_{eH}[MPa] Strain Gauge	R_{eH}[MPa] Extensometer	R_{eL} [MPa] machine	R_{eL} [MPa] Strain Gauge	R_{eL} [MPa] Extensometer
1-h-1	0.01	375.15	396.76	377.23	396.10	396.35	396.19	380.21	379.18	379.09
1-h-2	0.01	330.10	330.35	330.78	362.15	362.78	361.23	335.27	335.10	335.96
1-h-3	0.01	330.15	3320.10	320.37	355.05	355.62	355.10	323.07	322.08	322.21
2-h-4	0.01	370.12	-	375.10	381.12	-	381.17	374.10	-	376.11
2-h-5	0.01	350.15	-	335.55	361.05	-	361.72	340.11	-	340.81
3-h-1	0.01	370.12	-	375.10	381.12	-	381.17	374.10	-	376.11
1-v-1	0.01	348.10	336.25	330.87	358.56	358.56	358.56	349.10	336.25	332.07
1-v-2	0.01	324.12	-	330.19	339.10	-	379.93	324.04	-	330.10
2-v-1	0.01	335.10	-	336.14	358.21	-	357.17	334.29	-	333.04
1-h-4	0.02	340.01	335.13	335.13	351.00	351.00	351.00	330.10	330.10	330.10
2-h-1	0.02	355.10	350.00	350.00	367.10	367.10	367.10	343.18	346.15	346.15
2-h-2	0.02	346.22	-	343.04	352.00	-	352.00	340.08	-	340.08
3-h-2	0.02	335.22	-	338.17	353.23	-	353.37	340.07	-	340.07
3-h-3	0.02	350.10	-	340.19	366.04	-	366.04	340.14	-	340.14
1-v-4	0.02	370.00	371.12	372.33	397.11	394.15	395.10	364.18	367.15	370.12
1-v-5	0.02	375.00	378.08	378.08	397.26	394.12	395.14	364.00	371.04	357.00
1-h-5	0.03	340.10	340.00	330.00	370.10	370.10	370.10	335.21	332.11	333.00
2-h-3	0.03	337.00	333.41	337.48	365.68	365.41	365.20	334.07	333.00	332.07
3-h-4	0.03	335.00	350.10	345.12	370.00	370.10	370.10	335.21	332.11	333.00
3-h-5	0.03	345.10	-	337.18	370.00	-	370.00	346.23	-	345.20
2-v-2	0.03	351.00	355.04	355.04	362.43	362.23	363.51	351.00	354.10	354.10
2-v-3	0.03	351.00	353.12	354.15	362.12	362.58	363.98	351.48	352.11	353.41
3-v-4	0.03	350.00	-	350.11	346.11	-	343.15	340.21	-	342.52

Table Annex-B.3: Sample of data analysis

		R_m [MPa]			F_m[kN]			E [GPa]		
Sample No.	Speed mm/s	machine	Strain Gauge	Extensometer	machine	Strain Gauge	Extensometer	machine	Strain Gauge	Extensometer
1-h-1	0.01	501.10	-	501.10	80.20	-	8.20	-	209.03	201.15
1-h-2	0.01	501.20	-	501.20	75.39	-	75.39	-	205.35	195.78
1-h-3	0.01	456.15	-	456.61	73.06	-	73.06	-	196.71	199.05
2-h-4	0.01	480.68	-	480.68	76.91	-	76.91	-	-	194.17
2-h-5	0.01	472.43	-	472.43	75.59	-	75.59	-	-	199.23
3-h-1	0.01	488.125	-	488.12	78.16	-	78.16	-	-	195.22
1-v-1	0.01	458.56	-	458.56	74.40	-	74.40	-	200.15	194.28
1-v-2	0.01	470.93	-	470.93	75.35	-	75.35	-	-	199.16
2-v-1	0.01	470.68	-	470.68	75.31	-	75.31	-	-	201.04
1-h-4	0.02	464.68	-	464.68	74.35	-	74.35	-	192.75	202.13
2-h-1	0.02	473.18	-	473.18	75.71	-	75.71	-	208.15	203.11
2-h-2	0.02	470.37	-	470.37	75.26	-	75.26	-	-	198.13
3-h-2	0.02	470.25	-	470.25	75.24	-	75.24	-	-	199.03
3-h-3	0.02	474.37	-	474.37	75.90	-	75.90	-	-	197.32
1-v-4	0.02	491.25	-	491.25	78.60	-	78.60	-	211.59	192.22
1-v-5	0.02	481.00	-	481.00	76.99	-	76.99	-	209.16	196.27
1-h-5	0.03	459.31	-	459.31	73.49	-	73.49	-	199.78	196.83
2-h-3	0.03	462.00	-	462.00	73.93	-	73.93	-	202.16	197.27
3-h-4	0.03	480.56	-	480.56	76.89	-	76.89	-	-	202.08
3-h-5	0.03	480.00	-	480.00	76.80	-	76.80	-	-	200.13
2-v-2	0.03	487.87	-	487.87	78.93	-	78.93	-	217.00	193.17
2-v-3	0.03	480.68	-	480.68	76.91	-	76.91	-	201.62	192.18
3-v-4	0.03	481.62	-	481.62	77.06	-	77.06	-	-	209.05

Table Annex-B.4: Combined Uncertainty of Compressive Strength Measurement

Measurement Uncertainty Source	Source	Uncer.	Dis. Type	U	Dist Factor	Std. uncer. $u(x)_i$	Sen. Cof. c_i	u_ic_i	$u_i(y)^2$
Load (kN)	testing	u_p	Rect.	2.58	1.73	1.11	0.94	1.04	1.088
	reading	u_m	Rect.	0.19	1.73	0.11	0.12	0.01	0.0001
	eccentricity	u_e	Rect.	0.50	1.73	0.28	0.12	0.03	0.001
	cap angle	u_{cap}	Rect.	0.065	1.73	0.037	0.12	0.004	0.000002
	slenderness	u_s	Rect.	0.00	1.73	0.00	0.12	0.00	0.00
Diameter (mm)	vernier	u_{d1}	Rect.	0.10	1.73	0.577	-0.650	-0.375	0.140
Diameter (mm)	vernier	u_{d2}	Rect.	0.10	1.73	0.577	- 0.650	-0.375	0.140
Constant	pi rounding	u_{pl}	Rect.	0.0004	1.73	0.0002	- 7.786	-0.0018	0.000003
Strength (MPa)	pacer	u_{rep}	Rect.	0.025	1.73	0.15	-1.00	0.015	0.0002
								$u_c(y)$	1.159
								k	2.00
								U_{95}	2.318

$$f_c = \left[\frac{(X \pm u_p \pm u_m \pm u_e \pm u_{cap} \pm u_s)}{(\pi \pm u_{pi}) * ((d_1 \pm u_{d1} + d2 \pm u_{d2})/2)^2 /4)} \right] \pm u_{rep}$$

u_p - the uncertainty of the measured load taken as ±0.5 kN - rectangular distribution

u_m - the uncertainty associated with reading the testing machine ±0.5 kN - rectangular distribution

u_e - the uncertainty due to eccentric centering taken as ±0.258 kN - rectangular distribution

u_{cap} - the uncertainty associated with the angle of loading ± 0.0652 kN - rectangular distribution

u_s - the uncertainty associated with the slenderness ratio = 0.00 - rectangular distribution

u_{pi} - the uncertainty of the constant pi due to rounding ± 0.0004 - rectangular distribution

u_{d1} -the uncertainty associated with the measurement of diameter 1 ±0.1 mm - rectangular distribution

u_{d2} -the uncertainty associated with the measurement of diameter 2 ±0.1 mm - rectangular distribution

The measured compressive strength of concrete is 36.84 MPa with an expanded measurement uncertainty of ±2.318 MPa at a confidence level of 95% and with a nominal coverage factor of 2.The uncertainty of measurement value shown does not include any estimate of the effects associated with sampling or field and lab curing. Test results should be assessed using precision in terms of repeatability and reproducibility, measurement uncertainty and effects of sampling and curing.

The basis of the estimates of uncertainty and the estimate of value are shown above. Assuming that individual uncertainty sources are uncorrelated.

Solemn Declaration

I do solemnly declare that I have made this work without undue assistance from third parties and without the use of other than the referenced sources. From other sources directly or indirectly acquired data and concepts are identified by referencing the source.

Other people were not involved in the development of content and material of the present work. In particular, I have not used any paid help of mediation or counseling services (promotion consultants or other persons). No one has received directly or indirect monetary benefits from me for any work in connection with the content of the submitted dissertation.

The work has been neither submitted to any other Examining Authority in Germany nor abroad in the same or similar style. I certify on my honor that I have said the whole truth and that I concealed nothing.

Weimar, July 2015 Hem Bahadur Motra

Publications of the Autor

International reviewed Journals:

1. Keitel, K; Jung, B.; Motra, H.B.; Stutz, H.: Quality Assessment of Coupled Partial Models Considering Soil-Structure Coupling. In: *Engineering Structures* 59 (2014), 565-573.
2. Motra, H.B.; Hildebrand, J; Dimmig-Osburg, A.: Influence of specimen dimensions and orientations on the tensile properties of structural steel. In: *Material Testing*, Vol. 56, No. 11-12, (2014), 929-936.
3. Motra, H.B.; Hildebrand, J; Dimmig-Osburg, A.: Assessment of Strain Measurement Techniques to Characterize Mechanical Properties of Structural Steel. In: *Engineering Science and Technology, an International Journal*, Vol. 17 (2014), 260-269.
4. Motra, H.B.; Dimmig-Osburg, A.; Hildebrand, J.: Quality assessment of models with an application to cyclic creep prediction of concrete. In: *International Journal of Reliability and Safety*, Vol. 8, Nos. 2/3/4, (2014), 262-283.
5. Motra, H.B.; Dimmig-Osburg, A.; Hildebrand, J.: Evaluation of Experimental Measurement Uncertainty in Engineering Properties of PCC Samples. In: *Journal of Civil Engineering Research*, Vol. 3 No.3, (2013), 104-113.
6. Motra, H.B.; Hildebrand, J; Osburg, A.; Stutz, H.: A probabilistic method for quality evaluation of experiments in Civil Engineering. In: *Measurement, Journal of the International Measurement Confederation (IMEKO)*, under Review.
7. Scheiber, F.; Motra, H.B.; Legatiuk, D.; Werner, F.: Uncertainty-based Evaluation and Coupling of Mathematical and Physical Models. In: *Probabilistic Engineering Mechanics*, under Review.
8. Motra, H.B.; Hildebrand, J; Wuttke, F.: Monte Carlo method for the evaluation of measurement uncertainty: application to determine material properties. In: *Probabilistic Engineering Mechanics*, under Review.
9. Motra, H.B., Stutz, H., Wuttke, F. : Quality assessment of soil bearing capacity factor models of shallow foundations. In: *Soils and Foundations*, under Review.

National Journal:

1. Motra, H.B.; Dimmig-Osburg, A.; Hildebrand, J.: Quality assessment of strain measurement in concrete structures. In: *Bautechnik-Special Edition, Model quality*, 90(2013), 69-75.
2. Keitel, H.; Stutz, H.; Jung, B.; Motra, H.B.: Prognosequalität eines Gesamtmodells: Einfluss verschiedener Kopplungsszenarien auf die Interaktion Struktur-Boden. In: *Bautechnik-Special Edition, Model quality* 90(2013), 19-25.
3. Scheiber, F.; Motra, H.B.: Tragwerksmonitoring und numerische Simulation. In: *Bautechnik-Special Edition, Model quality* 90(2013), 63-68.

International Conference Papers:

1. Motra, H.B.; Hildebrand, J.; Dimmig-Osburg, A; Lahmer, T.: Reliability of Measurement Results for Materials Testing. In: *Proceedings of 12th International Probabilistic Workshop, IPW2014*, 4th - 5th November 2014, Weimar/Germany, ISBN: 978-3-95773-183-8, 228-237.

2. Scheiber, F.; Motra, H.B.; Legatiuk, D.; Werner, F.: Model Coupling in Structural Engineering Application. In: *Proceedings of 12th International Probabilistic Workshop, IPW2014*, 4th - 5th November 2014, Weimar/Germany, ISBN: 978-3-95773-183-8, 283-292.

3. Scheiber, F., Motra, H.B.: Ansatz zur hybriden Modellierung im Konstruktiven Ingenieurbau - Bewertete Kopplung von mathematischen und physikalischen Modell. In: *Proceedings of 7th symposium Experimentelle Untersuchungen von Baukonstruktionen*, Technische Universität Dresden/Germany, 2013, ISSN 1613-6934, 21-33.

4. Motra, H.B.; Dimmig-Osburg, A.; Hildebrand, J.: Uncertainty quantification on creep deflection of concrete beam subjected to cyclic loading. In: *Proceedings of 11th International Conference on Structural Safety & Reliability,Columbia University*, New York/USA, 16th - 20th June 2013, Taylor & Francis Group, London, ISBN 978-1-138-00086-5, 5141-5147.

5. Motra, H.B.; Dimmig-Osburg, A.; Hildebrand, J.: Influence of Measurement Uncertainties on Results of Creep Prediction of Concrete under Cyclic Loading. In: *Proceedings of 8th International Conference on Fracture Mechanics of Concrete and Concrete Structures*, 10th - 14th March 2013, Toledo/Spain, ISBN 978-84-941004-1-3, 805-814.

6. Motra, H.B.; Dimmig-Osburg, A.; Hildebrand, J.: Probabilistic assessment of concrete creep models under repeated loading with correlated and uncorrelated input variables. In: *Proceedings of 10th International Probabilistic Workshop*, Stuttgart/Germany, 15th - 16th November 2012, ISBN 978-3-921837-67-2, 285-301.